Topics: Nucleosynthesis and stellar evolution, pt. 2

Reading:

- Read LeBlanc, Ch. 6, the second half (from p. 245 through the end on p. 281).
- Take a look at the chart of nuclides linked on the website.

Summary of work to be produced:

- There are two warmup problems, due on Thursday at 5 pm: QW1 and QW9.
- Bring solutions to seminar on Friday for all the (non-warm-up) numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope:

We'll start by reviewing the evolution of a 1 solar mass star – Fig. 6.13 and then discuss the additional interesting complexities of the evolution of more massive stars.

End states (white dwarfs, neutron stars, black holes) will be a focus. Mass transfer in binary systems and observable phenomena (supernovae, novae) and heavy element and isotope nucleosynthesis (r- and s-process) are also things we'll discuss. We have a mix of observational and theoretical problems.

And there are a few topics we'll discuss, with a student presenting an explanation (of a figure or derivation) at the board. More about that on Monday or Tuesday.

Questions etc.:

Recreate/redraw/scan-annotate Fig. 6.13 and write down a sentence or two about each stage of the star's evolution in the HR diagram. Note specifically what fusion reactions are powering the luminosity and whether they're taking place in the core or a shell(s). You should be able to identify timescales as well – which stages are ones where the star's properties are changing relatively rapidly and which ones are quite steady-state? Refer back to other figures, nuclear reactions, and descriptions in the text (e.g. what does Fig. 6.9 on p. 237 have to do with a particular stage of the star's evolution?).

For massive stars, you should be able to describe how all the heavy elements built up over the star's life break down as the star collapses, ultimately producing a star made almost entirely of neutrons (under some circumstances). Highly endothermic reactions are quite common when a star is collapsing and there's lots of gravitational energy available. (What's the gravitational potential energy for a typical neutron star? How does it compare to the total luminosity of a massive star emitted over its entire lifetime? Or the Sun? Or the rest-mass energy of the neutron star?)

QW1 Problem 6.8 (parallax) – start with the full trigonometric expression relating the observed shift of the star's position with respect to the background stars to the star's distance from the Solar System and the size of the Earth's orbit. Begin with cgs (or mks) units. Use the small angle approximation and unit conversions (and the definition of the parallax angle) to derive the final expression.

Q2 Problem 6.10 (planetary nebula properties from observations)

Q3 Problem 6.11 (qualitative star cluster colors) and problem 6.15 (also qualitative question about star clusters) – provide a physical causes/explanations along with your answers.

Be prepared to discuss the step-by-step process of both a core-collapse supernova and a type Ia supernova. There are different types of core-collapse supernovae, differentiated by their light curves and spectra and – to the extent we can tell – by the physical processes.

Radioactive decay of heavy elements/isotopes produces in the explosion powers some of the phases of the light curves shown in the figures toward the end of the chapter.

Q4 Problem 6.12 (black hole density)

Q5 Problem 6.14 (supermassive black hole measurements – of an old-fashioned sort, compared to this week's news!)

The s- and r- processes and where different elements and isotopes come from is encapsulated in Fig. 6.36 on p. 274 (there are more complete and attractive diagrams linked from the website). Be prepared to explain what's going on here! Recall beta decay.

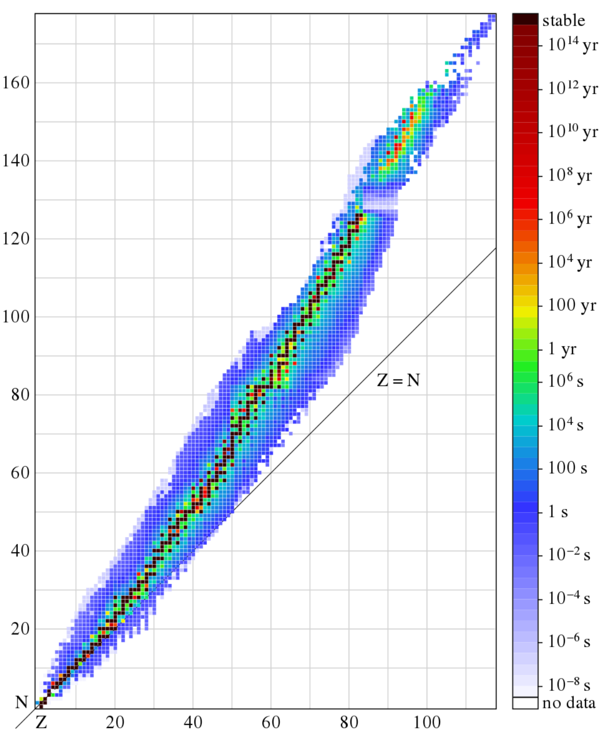

Fig. 1 Chart of every possible isotope, including the ones that actually exist, with their decay half-lives color coded.

Fig. 6.38 on p. 279 encapsulates a lot of important physics that goes into computing the rates of nuclear reactions. Be ready to explain the figure, especially the physics behind the two contributing/competing factors represented by the darker solid lines in the figure.

The rest of the problems are from Ostlie and Carroll – it's on reserve in Cornell, and I will soon put a scan of them on the website, though you'll want to consult the material in the actual chapter for some of them.

Q6 Problem 16.1 in O&C (measurable properties of a real white dwarf).

Q7 Problem 16.4 in O&C. You should compute the radiation and gas pressure from the properties given. You could do the same for the degeneracy pressure, assuming non-relativistic degeneracy, but the actual central pressure of Sirius B is given near the end of section 3 of chapter 16 in Ostlie and Carroll.

Q8 Problem 16.11 in O&C (tidal forces near a neutron star).

QW9 Problem 16.14 in O&C (if the Sun collapsed to a neutron star – rotation and magnetic field). Do both parts and then also compute the centrifugal force ratioed to the gravitational force for that angular-momentum-conserving solar-neutron-star.

Q10 Problem 16.18 in O&C (pulsar spindown) – can make your head hurt a bit thinking about which time is which.

Q11 Problem 16.6 in O&C (relativistic degeneracy pressure). I'm including the problem from last week, below. We can go over it before going over this.

Be prepared to discuss the *Chandrasekhar limit*.

Q8-wk11 (this question is an elaboration of what I did at the board during break in week 10) – it is from last week; I include it here for reference.

Note that LeBlanc just states the expression for the degenerate equation of state, and instead spends more time on the density of states in phase space and the maximum number of particles that can occupy a given portion of phase space. Once that density (eq. 5.136) is exceeded, degeneracy pressure is significant.

After reading about the pressure integral, I hope you're convinced that generically (i.e. not just for a perfect gas), the pressure can be approximated by $P = npv/3$, where n is the number density of particles, p is their momentum, and v is their velocity. A full expression involves an integral over the distribution of particle momenta (eq. 10.8 in Ostlie and Carroll) – and indeed, that's how you can derive the perfect gas law from an assumed Maxwellian particle velocity distribution). But for this problem, let's just approximate the pressure as $P = npv/3$ and assume that all particles have the same momentum (and mass, and therefore the same velocity).

Please do a dimensional analysis to make sure that npv has units of pressure.

OK – using this approximation, you are going to derive the equation of state for a non-relativistic degenerate gas (eqns. 5.137, 138). Do it as follows:

(a) Assume that the electrons are in a regular cubic lattice separated from their neighbors by a distance, x. What is the relationship between the number density, n, and x?

(b) Use the Heisenberg uncertainty principle to relate the momentum of the particles to their spatial confinement, x and thus to n.

(c) Use the classical expression that relates velocity and momentum to relate the velocity in the pressure equation to momentum and thus to the particle number density.

(d) Put it all together to derive eq. 5.137 including finding an expression for the constant of proportionality.

(e) Convert it to an expression in terms of mass density to derive eq. 5.138, again, including the constant of proportionality.

(f) Why is electron degeneracy pressure greater than neutron degeneracy pressure for a given mass density?