Topics: Radiation transport, part 1

Reading:

- LeBlanc, Ch. 3 (the first half, from the beginning of the chapter through the end of p. 89)
- Also read section 11 of Ch. 3 (pp. 99 - 100)

Summary of work to be produced:

- Hand in your solutions to the warm-up questions (QW4 and QW5) by Thursday at 12:30 pm in the box on the wall outside my office.
- Bring solutions to seminar on Friday for all the (non-warm-up) numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope: Radiative transfer has a certain mathematical complexity to it, but as always, focusing on a basic physics understanding and making sure you're okay with definitions of important quantities are the key. There's a lot of useful and interesting information in the reading for this week. The book problems cover/reinforce only a small portion of that material. I've invented a few additional problems to help you focus on the basic physics. But more than most weeks, it'll be important this week for you all to ask questions about the material during the seminar.

Questions etc.:

Make sure you're comfortable with spherical polar coordinates, solid angle, specific intensity, and as you read about them, the definitions and units of k, j, and S, the source function. You're familiar with radiation transport in a purely absorbing medium ($\frac{I}{I_o} = e^{-\tau}$), but now (the little bit of extra reading in section 11) you're getting the full version that includes emission by the medium, too (though scattering, largely, will have to wait).

On p. 73, for example, there's some vitally important stuff – make sure you can explain where the $\sin\theta$ comes from in the definition of flux. And get comfortable with the substitution $u = \cos\theta$.

Q1 Compare the number of photons entering your eye each second from a 100 W bulb 1 meter in front of your face to the number of photons emitted by the inside of your eye due to its non-zero temperature. Take your eye to be a hollow sphere with a radius of 1.5 cm and a temperature of $T = 310$ K that emits like a blackbody. Assume that all the photons it emits have the same wavelength (that of the peak of the blackbody) and similarly, assume all of the light bulb's photons have a wavelength of $\lambda = 5000$ Å. Take your pupil to have a radius of 0.2 cm. Comment on why you see the lightbulb but not your eye's own thermal emission.

Q2 Prove that the mean free path for a photon through a medium of density ρ and opacity k_ν is given by $\ell_{\rm mfp} = 1/\rho k_\nu$. You can do this by thinking of $I/I_o = e^{-\tau_\nu}$ as the probability that a photon will travel a distance x before being absorbed, where $\tau_\nu = \rho k_\nu x$. Note that the mean free path can be thought of as the expectation value of the path length, x, traveled by a photon, or the average value of x given the probability function $e^{-\tau}$. (Get in touch if this probability/expectation value stuff is completely unfamiliar to you.)

Q3 H^- opacity is the dominant source of opacity in the Sun's photosphere (in the optical part of the spectrum). Compute the ratio of H^- ions to neutral H atoms in the Sun's photosphere (you can use the Sun's effective temperature and take the electron density to be 10^{14} cm^{-3}). Note that H^- has no excited states (so its partition function is $U = 1$, given that it has two electrons that must have opposite spins). H^- provides bound-free opacity in the optical part of the spectrum via photoionization. Neutral H can also provide bound-free opacity in the optical, but not via photoionization out of the ground state (or "fundamental level"). What is the lowest level of neutral hydrogen for which photoionization can be accomplished by an optical photon? Compute the fraction of neutral hydrogen atoms in that excited state in the Sun. Is the number density of neutral H in that level more or less than that of H^- ions?

QW4 Compute the solid angle of a circular object that has an angular radius of 0.25 degrees (which is the size of the Moon or the Sun). Hint: Set up a spherical polar coordinate system (Fig. 3.4) with the z-axis pointing at the center of the circular object. Express your answer in both steradians and square degrees. What fraction of the whole sky (4π steradians) is that? How good is the small angle approximation for a solid angle of this size? To address that, compare the solid angle you calculated to the approximation that the object is a circle of angular radius 0.25 degrees and the solid angle is analogous to the area of a circle with that radius.

QW5 Compute the mean intensity, J, and flux, F, measured by an observer immersed in a uniform radiation field of intensity, I_{o}. Also, compute the flux you'd measure in a radiation field that's uniform but fills only one hemisphere (so, something like what you'd measure with a detector placed on the ground on a day that's uniformly overcast (the sky is a bright gray everywhere). The integrals are pretty simple, but please be careful and show your work. This gets at the $\cos\theta$ factor – comment on why, if a hemisphere represents 2π steradians, the answer for this last part is *not* $2\pi I_{\mathrm{o}}$.

Be ready to go over ex. 3.1 on p. 74, showing that for a star (or other spherical, uniform emitter of radiation - specific intensity is uniform across its surface), the flux emanating from its surface is just π times the specific intensity.

Q6 Problem 3.2. Note: There may be a typo in the statement of the problem in the book. The source function should be $a_\nu + b_\nu \tau^2$. That's tau *squared*.

Q7 Problem 3.3.

Q8 Problem 3.4.