Topics: Solutions to the Friedmann equation – radiation-, matter-, and Λ-dominated universes – dark matter

Reading:

- Read section 2 of Ch. 19 (pp. 439-43).
- Finish reading section 1 of Ch. 24 (pp. 555-58).

Summary of work to submit:

- Nothing to hand in for Tuesday's class.

Overview:

We talked about "flat rotation curves" of spiral galaxies as evidence for dark matter, in class on Thursday. Now we'll read about it in Ch. 19. Please think hard about the argument that the *shape* of the rotation curve (e.g. in schematic Fig. 19.7) tells us how the matter creating the gravity that determines the orbits of stars and gas clouds in galaxies is distributed. And, relatedly, how flat rotation curves imply that matter is much less centrally concentrated in galaxies than stars are. Dark matter exists in other environments too – notably in galaxy clusters, on large scales; and there, too, it's faster-than-expected velocities (of galaxies in this case) that tell us that there's more matter than we'd expect given the light we see coming from stars. Dark matter is everywhere, but it is not smoothly distributed; it clusters due to its own gravity. While dark matter's existence is notable, and while we know it's not baryonic, it does have gravity and so contributes to $\Omega_{\rm m}$. The matter, radiation, and cosmological constant (Λ, expressed as a fraction of the critical energy density, as Ω_Λ) all affect the expansion, but in different ways. In the reading from Ch. 24 we can see how $a(t)$ can be solved for in the approximations that one of these three components dominates. This enables us to compute the acceleration (and draw conclusions about what type of universe will expand forever and which won't) and compute quantities like the horizon distance and the age relative to the Hubble time in the cases where one component dominates. Finally, though there's not an analytic expression, we can look at the history and future of the scale factor in detail, know just the three values for the three Omegas (Fig. 24.3).

Commentary on the reading, viewing, and other preparation:

Stars and even gas clouds in spiral galaxies like the Milky Way and Andromeda orbit the centers of these galaxies in more-or-less circular orbits. On page 440, Kepler's third law is applied to the Solar System orbiting the Milky Way's center to weigh the Milky Way – or, importantly, just the mass *interior to* the Solar System's orbit.

See how eq. 19.12 is just our familiar equality of centripetal and gravitational acceleration, acknowledging that the relevant mass is the mass enclosed, $M(R)$? Fig. 19.7 is a schematic version of the Milky Way's flat rotation curve. Compare it to the actual one shown on p. 459 and to the Andromeda curve. And importantly, compare it to the expected Keplerian $r^{-0.5}$ curve. At the bottom of that page you see explicitly how much more mass there is beyond the Sun (as derived from the rotation curve). And at the top of p. 442, it's noted that the specific form of the rotation curve (v = constant) implies a specific form of $M(R)$ (do you see how/why??) and thus of density ($\rho \propto R^{-2}$). Can you see how that follows from the mass-enclosed vs. radius relation?

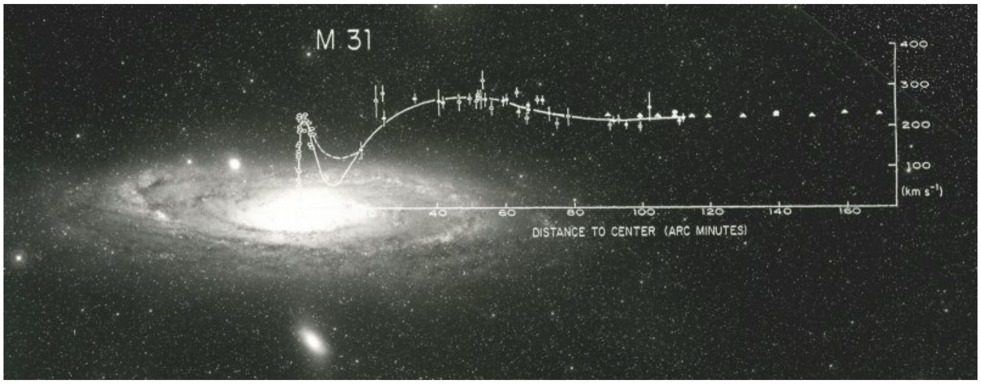

Fig. 1 The measured rotation curve (from numerous Doppler shift measurements) of the Andromeda Galaxy (M31), superimposed and to scale on an image of the galaxy itself. Note that if the gravitational mass were distributed like the light in the galaxy, once you look beyond the bright nucleus of the galaxy, you'd expect the orbital velocity to fall off like the square root of the distance. But it is flat.

The discussion of what the dark matter could be is a little outdated, though it's interesting to consider these alternatives. We have good evidence that the dark matter is (largely) composed of WIMPs. (We'll see some of that evidence during week 14, when we learn about Big Bang nucleosynthesis.)

Finally note that gravitational lensing is a good (but complicated) way to look for dark matter. The discussion in the reading of the search for individual objects floating around our galaxy's "halo" (spherical distribution of matter in which the flat, spiral disk is embedded) discusses "microlensing" and we saw in class last week how stronger (plain old) lensing can be used to infer the amount of mass in a galaxy cluster that distorts the light of background galaxies. ...there is lots of evidence, in many environments, of dark matter, and in large quantities.

This is a good time to review the mass-energy census – see the table showing the consensus model on p. 554.

The Friedmann equation – do you accept the version (and interpretation) shown in eq. 23.68? And then the version in eq. 23.71? Can you name and describe the meaning/interpretation of each term? Get in touch before class if you have questions!

Remind yourself of the critical energy density (eq. 23.74) and physics/logic of the scale-factor dependencies of each of the three component energy densities (text at the beginning of Ch. 24, including the first two numbered equations of that chapter).

...and, how do we get from eq. 23.71 to eq. 24.13. Get comfortable with that last version!

Bottom of p. 555 – method for finding when different components had equal energy densities (and thus the ranges of the scale factor over which any one particular component dominates).

OK! Now see how if you have only one kind of matter/energy that's important, with one particular scale factor dependence, and with a flat universe, you can just take the square root of the Friedmann equation and have a single term on the r.h.s. and just the first derivative on the scale factor on the l.h.s.? And you can then easily integrate the equation to find $a(t)$ (and thus, for example, the sign – and even the magnitude – of the acceleration).

Can you see, for example, how the age of a flat, matter-dominated universe is two-thirds of the Hubble time? (That's a challenge – see if you can do it!) In the late 1980s this was a problem, because the universe did seem to be (close to) flat and matter dominated and thus kind of young; like, younger than the oldest stars. And that isn't so good. Do you then, see, at least qualitatively, how having some Λ makes the universe older (if the expansion is now speeding up, it must have been slower in the past and therefore taken more time to expand to its current size)?

And in general, whether the expansion is decelerating or accelerating depends on the scale-factor dependence of the dominant mass-energy component (see particularly, footnote 2 on p. 557).

Finally for this section, pretty wild that a pure cosmological constant universe expands exponentially (eqs. 24.27, .28).