

**Topics:** Finishing hydrostatic equilibrium and then on to spectral types, luminosity classes, and the Hertzsprung-Russell diagram

**Reading:**

- Review the barometric equation and scale height (pp. 215-16)
- Review section 1 of Ch. 14 and then read the rest of Ch. 14
- Read/look over the slides linked from the class website:  
[http://astro.swarthmore.edu/astro16/reading/A16\\_class12\\_pre.pdf](http://astro.swarthmore.edu/astro16/reading/A16_class12_pre.pdf)

**Summary of work to submit:**

- Nothing to submit for Thursday's class

HSEQ for a geometrically thin, isothermal atmosphere leads to an exponential density (or pressure) structure (the barometric equation) with a characteristic length scale (the scale height) – the scale over which the density or pressure changes by a factor of  $e = 2.71$ . This scale height depends on the strength of gravity, the temperature, and the mean molecular weight.

Once we have the density structure of an atmosphere, we can compute the optical depth at any wavelength for which we know the atomic cross sections. (And note: which cross sections are relevant depends on the ionization state of each element, which depends on the density, so we really do need to know the density of a stellar atmosphere in order to understand it). And that enables us to understand the spectra we see from stars and then to *classify* those spectra based on the relative strengths of various absorption lines (§14.2) and also we can classify stellar spectra based on the *widths* of certain absorption lines, which provides information about the pressure of the gas in the star's atmosphere, and thus on the strength of gravity at the star's surface. This second dimension of classification is referred to as the *luminosity class* and it differentiates normal ("main sequence") stars from giants and supergiants (§14.3). By plotting the luminosity of stars vs. their surface temperature (the Hertzsprung-Russell diagram, or HRD) we see several important trends, including the most fundamental one: that main sequence stars show a strong correlation between luminosity and surface temperature (hotter stars are more luminous). We'll find out later that the underlying, causal factor is the star's mass – more massive stars are more luminous and this causes them to have higher surface temperatures. But for now – for Ch. 14 – the HRD is presented as phenomenological. The physics behind it will be introduced in Ch. 15, which we'll read after break.

**Commentary on the reading:**

HSEQ review: The *scale height*, described in Ch. 9, represents the distance over which the pressure in an atmosphere changes by a factor of  $e$ . And it is dependent on the physical properties (specifically,  $g$ ,  $\mu$ , and  $T$  – think about why  $H$  is bigger if  $T$  is bigger but is inversely proportional to the other two quantities). Why is the Sun's atmospheric scale height bigger than the Earth's?

Section 2 of Ch. 14 shifts gears – given a stellar atmosphere in HSEQ, what does its spectrum look like and what trends are there? Think about how answering this question involves solving the Saha and Boltzmann equations (like we did for hydrogen Balmer absorption in Ch. 5 – Fig. 5.13 – you should think about that figure as you read the text on p. 340). Also, take a look at (click on) the leftmost image at the top of the class webpage, as you start to think about spectral subtypes.

And as noted in the reading bullet points, look over the slides I've put on the website. Do you see the hydrogen Balmer lines in the first image (or in the leftmost image at the top of the class website)? Do you see how they peak in strength at intermediate temperatures? Can you also see the Wien shift (from blackbody emission – the hotter spectra are brighter in the blue and the cooler ones brighter in the red)? And can you see some broader, molecular lines in the coolest stars' spectra?

On p. 340, the Saha-Boltzmann physics behind absorption line strengths is revisited (recall, again, Fig. 5.13). Note that the Saha and Boltzmann equations both have terms that look like  $e^{-E/kT}$ , where  $E$  is the ionization potential for Saha and the excitation energy for Boltzmann. So, that's why the strengths of absorption lines are functions of temperature. And why the spectral type sequence (which was discovered as simply an *empirical* classification scheme well before the physics was understood – much like Linneaus categorized living things into genus and species well before DNA was discovered) is really a temperature sequence, dependent on the surface temperature (or *effective temperature*) of a star. Star surface temperatures range over about a factor of 20 (p. 342) and *brown dwarfs* are cooler than stars but part of the same spectral classification system.

Luminosity class is also empirical – and depends on the *widths* of certain spectral lines – those whose widths are governed by pressure broadening rather than thermal broadening. The underlying physical property that controls the luminosity class is the surface gravity of a star (and thus the size, or radius of a star). So this classification allows us to distinguish more compact *main sequence* stars from giants and supergiants.

The Hertzsprung-Russell diagram (now about 100 years old) shows that star properties are far from random but rather are correlated. Perhaps most importantly, it shows how luminosity is directly related to surface temperature for main sequence stars. This spurs us to understand *why* this relationship holds. Note also that the x-axis on an HRD can be color or spectral type or surface temperature. And the y-axis can be absolute magnitude rather than luminosity. The radii of stars is encoded in an HRD, since it plots  $L$  and  $T$  and we know that  $L = 4\pi R^2\sigma T^4$ . Finally, note that when we put a bunch of stars on the HRD (i.e. when we measure their temperatures and luminosities) we find that there are many more dim, red stars than bright blue stars (left panel of Fig. 14.3).