Hand in your solutions by 9pm on Friday, October 13. You should put them in the lower box outside my office.

The *color* of a light source (like a star), broadly speaking, is governed by the relative amounts of long-wavelength (red) and short-wavelength (blue) light. But there are different ways to quantify "relative amount." In the first three questions we explore two ways to do this and assess which way – a difference-based method or a ratio-based method is better.

Problem 1

Here we explore a difference-based method, which essentially estimates the slope of a function between two points. To do this, compute a function (call it $A(T)$) that is defined as

$$A = \frac{B(6000, T) - B(4000, T)}{2000}, \tag{1}$$

where B is the Planck function (equal to I_λ in eq. 5.90; note that I_λ is a symbol that represents any specific intensity (per unit wavelength) whether the light is from a blackbody or not, but B or B_λ represents the specific intensity of the Planck function). Note also that we explicitly express B as a function of wavelength and temperature. In the plotting in the previous two problems, you fixed T at a particular value and plotted B as a function of λ. Here, you are fixing λ and letting B (and A) be a function of temperature, T. Plot $A(T)$ over the range 2600 K to 20,000 K. Make a well labeled plot with thoughtfully chosen axis ranges.

Note (just a point of information, not relevant to solving the problem) that 6000 Å is a relatively yellow wavelength, not quite red.

Problem 2

Now we'll explore a ratio-based method. To do this, compute a function (call it $C(T)$) that is defined as

$$C = \frac{B(6000, T)}{B(4000, T)}, \tag{2}$$

and plot it over the same temperature range. (Separate plot, though.)

Problem 3

Which diagnostic, $A(T)$ or $C(T)$ will be more useful for measuring the temperature of stars? Think about a couple of things – for one, you want to be able to measure A or C for a given star, which relies on knowing the intensity (or luminosity) from the star at two different wavelengths. But like luminosity, intensity isn't directly measured, flux is. To convert from flux to intensity (as with luminosity) you use the inverse square law and thus need to know the distance to get an exact value

Problem 3 continued on next page. . .

of the measured Planck function, B. Does either A or C not actually depend on knowing the distance (again, treat B like a luminosity noting that you measure flux and flux and luminosity are related through the inverse square law).

Another thing to consider is that for this color-temperature diagnostic to be useful, it can't vary wildly in value (since it's hard to measure huge ranges of most physical quantities) but it should vary somewhat as a function of temperature so that a particular measured color (A or C value) corresponds to a specific temperature rather than a huge range of temperatures.

Given these considerations, explain why you think one of the diagnostics is better than the other.

Problem 4

Consider a solid (spherical) object, at some distance, d, from a star with a luminosity, L. The object is heated by the starlight it absorbs and cooled by its own (assumed) blackbody radiation.

(**a**) Assuming thermal equilibrium – its temperature doesn't change with time – derive the relationship between the object's distance from the star and its temperature, T. Assume that all the starlight that it intercepts is absorbed and contributes to heating the object. You should assume that the area presented by the object is its two-dimensional cross-sectional area and that the area relevant for the reemission of the object's thermal radiation is its entire surface. The relationship you derive between T and d should depend only on L and physical constants.

(**b**) Evaluate the relationship you derived for an object orbiting the Sun at a distance of $d = 1$ AU.

(**c**) Comment on this temperature relative to the actual temperature of the Earth.

(**d**) What physics is left out of our simple model?

(**e**) Sketch the blackbody spectrum of the Sun measured at the Earth and the blackbody spectrum of the Earth (given the temperature you derived). Wouldn't you agree that the areas of under these two spectra should be the same? (Why?)

(**f**) Now, go back to the relationship you derived for $T(d)$ and – assuming the Sun radiates like a blackbody with temperature $T_{\rm sun}$ – express the Sun's luminosity in terms of its temperature and radius, $R_{\rm sun}$ and then reorganize your expression for the temperature of the object so that it's got $T/T_{\rm sun}$ on the left-hand side.

(**g**) Finally, can you express $(T/T_{\rm sun})^4$ in terms of the Sun's angular radius, $\theta_{\rm r}$ and then the Sun's *solid angle*, $\Omega_{\rm sun} = \pi\theta_{\rm r}^2$?

Problem 5

Ryden & Peterson problem 13.4 (p. 334).

Problem 6

Ryden & Peterson problem 13.6 (p. 335).

Problem 7

Ryden & Peterson problem 13.7 (p. 335). Answer the following questions, too:

If the orbital inclination were actually 45 degrees, then what would the orbital speeds of the two stars actually be (assume that their orbits are circular)? And what would their masses be? And would our estimate of the semi-major axis of the orbit be bigger or smaller if it turned out that the orbital inclination were 45 degrees instead of 90 degrees (but all the other information provided in the problem were still the same)? Note that there are three separate questions in this part of the problem.

Problem 8

What fraction of the mass of the Earth's atmosphere is between sea level and an altitude of 8 km?

Problem 9

Ryden & Peterson problem 13.2 (p. 334), but *only* part (b).