Astronomy 6 – Introductory Cosmology
Fall 2013

Homework #2
Due Wednesday, December 4 by 10am

This short assignment is about **dark matter**. The first, multipart problem is about spiral galaxy rotation curves and the dark matter in our own galaxy, the Milky Way. It covers both the observational evidence and properties and also some basic theoretical considerations. The second problem is essentially a reading guide for the *Physics Today* article about the Bullet Cluster.

Note that your answers to the two problems are due by 10 am on the day of our next class. I'll look them over, grade and comment on them, and give them back to you in class later that day, to aid in our class discussion.

The purpose of the homework is to help prepare you for class. Note that some of the parts of the two questions are pretty basic; but they're still worth answering explicitly and carefully to review and solidify your understanding of how the concepts fit together.

Please read the following paragraph!

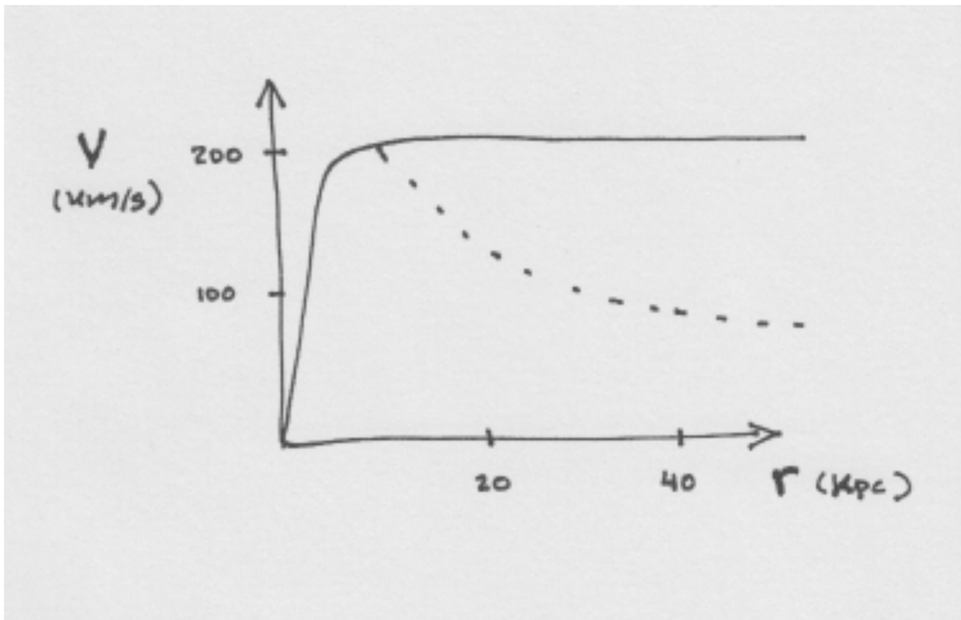
For full credit you must show your work, use units, and include explanations, sketches, and graphs whenever you think they might be useful. Clear writing (both figuratively and literally) is important. Note that useful physical and astronomical constants can be found in the appendices of the textbook. And for numerical problems in this problem set, two significant figures will be enough.

1. Stars and clouds of interstellar gas in spiral galaxies all orbit the center of their galaxies in basically circular orbits, governed by basic Newtonian physics, so that the velocity of any particular star or gas cloud is given by:

$$v = \sqrt{\frac{GM_r}{r}}$$

where v is the linear velocity in m/s, G is Newton's gravitational constant, r is the distance from the center of the galaxy (in meters), and M_r is the mass interior to radius, r (in kg). Note that the Solar System is about 8.5 kpc from the Milky Way's center. And that there are 3.1×10^{19} m in a kpc.

When the behavior of v as a function of r (the "rotation curve") is mapped out for the Milky Way, we find something like the solid line here:



Note that the dashed line shows the rotation curve the Galaxy would have if it were composed only of baryonic matter (i.e. if the stars and gas were all there were to the Milky Way).

a. Derive the equation on the previous page by equating centripetal force (description of uniform circular motion) with Newton's gravitational force (the cause of the circular motion for orbiting objects).

b. From the rotation curve plot, what is the total mass of the Milky Way considering the edge of the Galaxy to be at a radius of 50 kpc? Express your answer in both kg and in solar masses.

c. If you take the volume of the Milky Way to simply be a sphere with radius 50 kpc, what is the average density of the Milky Way?

Express your answer in both mks units and in terms of the critical density, so Ω_{MW}.

d. Why is the number you found in (c) for Ω_{MW} so much bigger than the average value for the Universe as a whole, $\Omega \sim 0.3$?

e. We can assume (simply based on the brightness distribution of the Galaxy) that the vast majority of the baryonic matter in the Milky Way is in the inner 10 kpc. Given that, what is the mass of the Milky Way's baryonic matter, based on the rotation curve plot? You may assume that baryonic matter totally dominates dark matter for r < 10 kpc and dark matter totally dominates beyond that. Of course, that's an oversimplification, but a decent first approximation. Express your answer in both kg and in solar masses. (Note that you can probably avoid some calculations by comparing this problem to part (b)).

f. What is then the mass of dark matter in the Milky Way (combining your results from (b) and (e))? Express your answer in both kg and in solar masses.

Note that this is just for the dark matter within 50 kpc. There's evidence that the dark matter "halo" of the Milky Way extends well beyond 50 kpc, perhaps out to the dark matter halo of the Andromeda Galaxy (a few hundred kpc away).

g. Switching gears a bit: From the figure, we can see that $v(r)$ is linear out to $r \sim 2$ kpc. Given that v is proportional to r in that inner region, what is the density as a function of radius in that region? (hint: find M_r first, then find ρ) Recall also that in class we showed that the flat rotation curve at larger radii implies that ρ is proportional to $1/r^2$.

Given the density proportional to $1/r^2$ result for the dark matter, and the total mass of dark matter you found in (f), we have a density of dark matter at $r = 10$ kpc of $\rho = 5 \times 10^{-22}$ kg m^{-3}. Within that radius we can assume that the dark matter density is constant (aside: note that a Navaro Frenk White profile - http://en.wikipedia.org/wiki/Galaxy_rotation_curve#Halo_density_profiles - is a more accurate description of the dark matter distribution in spiral galaxies, but our simple approximation is okay).

h. Taking the assumed-constant dark matter density, above, in the center of the Milky Way (ρ = 5 X 10^{-22} kg m^{-3}), what is the maximum particle density of the (still unidentified) dark matter particle assuming that its mass must be at least the mass of the Higgs Boson (125 GeV; see Table 23.1 and text surrounding it if you're unsure about these units)? If it were lighter, it would have been discovered in a particle accelerator already.

   Note that the number you found for the average baryonic matter mass density in the inner 10 kpc of the Milky Way corresponds to about a million protons per cubic meter, for comparison.

i. And switching gears one last time: A small minority of cosmologists and physicists are skeptical that dark matter exists, and instead suggest that the theory of gravity needs revising (and in some new theory, the regular, light-emitting, baryonic matter we see will explain the flat rotation curves without having to resort to dark matter). This is the MOND framework mentioned in the second paragraph of the *Physics Today* article. Now, we've seen in physics how classical theories are embedded in a more correct theory. And the correct theory usually holds in some high energy limit in which the classical theory fails, but the correct theory reduces to the classical theory in the low energy limit. Think about special relativity in this sense – very fast speeds require SR but at low speeds SR reduces to classical Newtonian dynamics.

   Given this, let's take a look at the key dynamical quantity, the acceleration (since gravity causes acceleration, and that leads to the orbital motion we measure). What is the acceleration of a star on the edge (r ~ 50 kpc) of the Milky Way, according to the rotation curve plot? Give your answer in mks units of acceleration, and also express it as a fraction of the gravitational acceleration on the surface of the Earth. So, do the strange, flat rotation curves occur where gravity is very strong or very weak?

2. Reread "Collision between galaxy clusters unveils striking evidence of dark matter" by Bertram Schwarzschild, from *Physics Today*. Answer the following questions and come to class prepared to discuss the article.

    a. What is the main component of baryonic matter in galaxy clusters and with what kind of telescope is it measured with in this Bullet Cluster observation?

    b. Why are the baryonic and dark matter *not* centered in the same location in the case of the Bullet Cluster? What happened to the main component of the baryonic matter when the two galaxy clusters passed through each other? What does the caption to Fig. 2 mean when it says, "…the collision retarded the plasma…"?

    c. Why doesn't the dark matter in each galaxy cluster also get "retarded by the collision"? Why didn't the galaxies themselves smash into each other when the clusters passed through each other?

    d. How is the total gravitational mass (baryonic plus dark matter) of each cluster measured?

    e. What do the two blue crosses in Fig. 1 signify? What about the pair of groups of three concentric bright white ovals? How does the fact that the blue crosses don't coincide with the locations of the white ovals show that most of the mass in the galaxy clusters is *not* baryonic matter?

    f. In the last full paragraph on page 22 the author says "The two peaks of the bimodal total-mass distribution coincide well with the two concentrations of galaxies that mark the cores of the recently collided clusters." Does that mean that the light-emitting matter (the stars whose light we see in the optical images of the galaxies in Fig. 1) contribute a significant fraction of the total mass? If not, why do the contours show that the bulk of the matter is where the galaxies are?

Also, a comment about terminology in the article:

- A "bow shock" is named after the front, or bow, of a boat. Think of the V-shaped pattern a moving boat makes in the water.