Astronomy 6 – Introductory Cosmology
Fall 2013

Homework #3: preliminary version 2
Due Thursday, December 12, by noon

This final assignment focuses on models of the universe and on cosmic acceleration.

For full credit you must show your work, use units, and include explanations, sketches, and graphs whenever you think they might be useful. Clear writing (both figuratively and literally) is important.

1. For the Newtonian, critical, matter-dominated universe ($\Omega_m = 1$, $\Omega_\Lambda = 0$), compute the horizon distance, ℓ_{hor}, in terms of H_o, the current value of the Hubble constant. Recall that the horizon distance is the proper distance, ℓ_p (note that these are meant to be a script "ell" – the letter between *k* and *m*) to an object whose light emitted just after the Big Bang is reaching us now.

    a. Compare your result to the Hubble distance – the age of the universe times the speed of light. Note that the age of the universe is not necessarily equal to the Hubble time, $t_H = 1/H_o$; that t_o is the current age of the universe in a system where $t = 0$ corresponds to the Big Bang; and that you've already computed the age of the universe for the Newtonian, critical universe in a previous assignment.

    b. Is this value less than, equal to, or greater than the horizon distance? Why is that so?

    c. Comment on whether the horizon distance in this universe is finite or infinite and whether it will always be so, and if it'll get bigger or smaller in the future. No long explanation is required, but do briefly justify your answer(s).

    Remember: we do *not* live in this universe.

    And **note**: you have computed a(t) for this universe already (in one of your *Mathematica*-based problems), but it may be easier to solve this problem if you start over with the version of the Friedmann equation expressed in eqn. (24.14) and use a time coordinate that has $t = 0$ at the Big Bang and $t = t_o$ now.

2. Similarly, you can derive an expression for the horizon distance in a critical, cosmological-constant-dominated, or Lambda-dominated, universe, ($\Omega_m = \Omega_r = 0$, $\Omega_\Lambda = 1$). For such an exponentially expanding universe without a Big Bang, there are some subtleties we'll have to deal with. You've already computed the evolution of the scale factor for this universe, too. You got an expression much like eqn. (24.27) in the book.

   Because there's no Big Bang in this universe, there's no completely obvious timescale and thus no natural $t = 0$ definition. Related to this, there *is* a particular length-scale built into this universe – that is, whatever the scale factor's value is at $t = 0$ defines a certain length scale. So, we are free to define whatever time-zero we want, and then that defines the scale factor at that time: $a(t = 0)$. It seems like a relatively natural choice of time-zero is one Hubble-time ago. In other words, we define the current time $t_o = 1/H_o$. And thus if we have for the scale factor in this universe:

$$a = a(t = 0)e^{H_o t}$$

   the value of $a(t = 0)$ can be found by noting that $a = 1$ at $t = t_o$.

   a. Find the numerical value of $a(t = 0)$ given our choice of $t_o = t_H$.

   b. Given this complete expression for $a(t)$, find an analytic expression for the horizon distance in this universe. Any integrals must be done by hand (show your work). Your answer should be in terms of c, H_o, and t_o.

   c. Comment on whether this expression is finite or infinite right now.

   d. What asymptotic value does the horizon distance approach in the distant future (as time goes to infinity)?

   e. What is the value of the Hubble parameter, $H(t)$ at times other than t_o in this universe (you might want to check your *Mathematica* notebook and associated assignment)? And does this universe keep expanding forever?

f. Argue that in this universe we will eventually have no other galaxies within our horizon (i.e. closer than the horizon distance).

g. Argue that even though this isn't exactly the universe we live in, that this will be our fate anyway (if the energy density of the cosmological constant, Λ, is indeed constant in time).