Astronomy 6 – Introductory Cosmology
Fall 2013

Scale Factor Project
First milestone, due Monday, November 18 by 2:30 pm

This is just a formalization of the assignment you had for Friday: to add the scale factor vs. time for the empty universe to my *Mathematica* notebook that already had the critical, Newtonian universe plotted. And you had computed that function – the scale factor vs. time for the critical, Newtonian universe in the previous homework problem set.

Now, I'd simply like you to hand in a neat and clear version of your derivation of $a(t)$ for the empty universe case, along with your modified *Mathematica* notebook, showing the two $a(t)$ functions.

In order to do this, you should start with the Newtonian (no cosmological constant, Λ) version of the Friedmann equation, eqn. 23.67 on p. 547.

For the critical case, we set $k = 0$, leaving only the first term on the right hand side. Then we expressed the density, ρ, as an explicit function of the scale factor, $\rho = \rho_0 a^{-3}$. Then we separated (all terms with a in them on the l.h.s.) and integrated, applying two boundary conditions in order to evaluate the constants that showed up in our solution.

In this case, for the empty universe, you'll do something analogous, but even simpler: since the universe is empty, the density is always zero and thus the first term on the r.h.s. is zero. Thus you essentially follow the same procedure but with the second term on the r.h.s. now being relevant. Separate, integrate, apply two boundary conditions.

Here are some more specific tips and guidelines:

Use $t - t_o$ as the independent variable (rather than time, t, itself). This way, $t = 0$ is now. While it would make sense to have $t = 0$ be the time of the big bang, since the universe will have a different age for different assumptions, the scale will be arbitrary once you start plotting more than one model. Better to have now be $t = 0$. Then the age of the universe is $-(t_{bb} - t_o)$, and that quantity is simply graphically read off your plot as the x-intercept. And $t_o = 0$, so all models go through (0,1) since $a(t{=}t_o) = 1$.

Don't ignore the $2k/r_o^2a^2$ term. To solve the empty universe, you have to find the value of $2k/r_o^2$. You do that by imposing boundary conditions: (1) $a = 1$ at $t - t_o = 0$ (now). And (2) $a = 0$ at $(t - t_o) = (t_{bb} - t_o)$.

This last condition will be especially useful when you remember that you already know that in the empty universe, the age of the universe is exactly equal to the Hubble time, $t_H = 1/H_o$.

Separately and for more complicated cases than the empty universe, it's generally a good idea to look to be substituting H_o for $8\pi G\rho_{crit}/3$.

And also to think about the energy and mass terms on the r.h.s. of the Friedmann equation in terms of their explicit dependence on the scale factor, a. In the empty case you'll find the a terms cancel out, but in other cases, they won't.