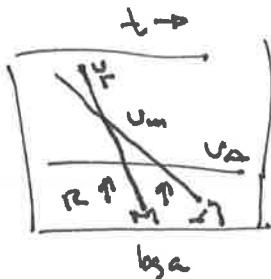


Fig 24.2



$$V_r \propto a^{-4}$$

$$V_m \propto a^{-3}$$

$$V_L \propto \text{const}$$

since $a(t_0) = 1$ $V_r = V_{r,0} a^{-4}$ etc.

\Rightarrow diff. dep. means diff. components will dominate in different range of scale factor, a

p. 553

How to normalize

~~Subtract~~ These - heights
of linear

e.g. the first two increasing

if $V_m = \text{constant}$ means
 a will eventually dominate

\rightarrow find intersection points
by setting, e.g.

$$V_r \text{ from } T_B \quad S_{L,r} \sim 5 \times 10^{-5}$$

$$V_m = V_m$$

$$V_m \text{ from census of matter: } S_{L,m} \sim 0.3 \\ \text{lensing, dynamics (gal. clusters)}$$

$$V_{\Lambda,0} = V_m a^{-3}$$

$$V_\Lambda \text{ from } H(t) \text{ measurements: } S_{L,\Lambda} \sim 0.7$$

$$a^3 = \frac{V_{m,0}}{V_{\Lambda,0}} = \frac{1}{3}$$

$$a = 0.7$$

\curvearrowright Accelerating
Expansion

what redshift?

$$\curvearrowright 1+z = 1/a$$

Also because V_r has the
steepest dep. on a ,
if a is small enough

 $V_r \gg V_m, V_\Lambda$ inthe early universe

framework: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} V(t) - \frac{K c^2}{r_{c,0}^2 a^2} + \frac{\Lambda}{3}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} (V_r(t) + V_m(t) + V_\Lambda(t)) - \frac{K c^2}{r_{c,0}^2 a^2}$$

P. 554 conservative model \rightarrow flat, $K = 0$

$$\therefore V_c = \frac{3H_c^2 c^2}{8\pi G}$$

$$H^2 = \left(\frac{8\pi G}{3c^2} \right) \left(\frac{V_{r,0}}{a^4} + \frac{V_{m,0}}{a^3} + V_{\Lambda,0} \right)$$

~~$$H^2 = \frac{H_0^2}{V_{c,0}} \left(\frac{V_{r,0}}{a^4} + \frac{V_{m,0}}{a^3} + V_{\Lambda} \right)$$~~

but since $\frac{V_{r,0}}{V_c} = \sum r_{r,0}$

$$\underbrace{\dot{a}^2}_{\sim a^2} H^2 = H_0^2 \left(\frac{\sum r_{r,0}}{a^2} + \frac{\sum m_{m,0}}{a} + \sum \Lambda_{\Lambda,0} a^2 \right)$$

$$\frac{da}{dt} = H_0 \sqrt{\frac{\sum r_{r,0}}{a^2} + \frac{\sum m_{m,0}}{a} + \sum \Lambda_{\Lambda,0} a^2} \quad \Longleftrightarrow \text{solve this diff. eq. to find } a(t)$$

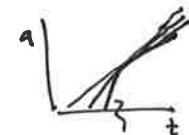
$a(t)$ in any, arbitrary flat universe

As you did, we can look at exact solutions when one component dominates — i.e. we can learn things

e.g. early universe: $\Sigma_{\zeta_0} \approx 1$ ($\Sigma_L = 1 \because \omega_r \gg u_m, u_n$ at early t)

you found:

$$a(t) = \sqrt{1 + 2H_0(t - t_0)}$$



p.556 using time-scale where $t_{bb} = \phi$ (rather than $t_0 - t_0$)

\therefore expanding in terms of Σ_{ζ_0}

$$a(t) = \sqrt{2\Sigma_{\zeta_0}^{1/2} H_0 t} \propto t^{1/2}$$

Recall
(p.545)

proper distance

$$cdt = adr$$

metric of spacetime
incl. expansion

$$c \frac{dt}{a} = dr$$

$$\int_{t_0}^{t_0} \frac{dt}{a} = l_p \sim$$



p.546 def'n of horizon distance:

\hookrightarrow proper distance of the farthest away thing we can — in principle — see

(distance of object whose light emitted at $t=0$
~~is~~ is just reaching us now)

$$l_p = c \int_{t_0}^{t_0} \frac{dt}{a(t)}$$

So, given a universe $(H_0, \Omega_m, \Omega_r, \Omega_\Lambda)$

we can solve Friedmann to get $a(t)$

explicitly; then we can compute l_p
to any object we observe.

ex. p. 946

assume $a = a_0 (t/t_0)^n$

$$l_{hor} = c \int_0^{t_0} \frac{dt}{a(t/t_0)^n} = c t_0^n \int_0^{t_0} \frac{dt}{t^n} = c t_0^n \frac{1}{1-n} t^{1-n} \Big|_0^{t_0}$$

$$= c t_0^n \cdot \frac{1}{1-n} (t_0^{1-n} - 0) = \frac{c t_0^n}{1-n} t_0^1 t_0^{-n}$$

$$l_{hor} = \frac{c t_0}{1-n}$$

"current
proper distance
to the
particle
horizon"



? we saw for $\Omega_r \approx 1$ $n = 1/2$ \therefore

$$l_{hor} = \frac{c t_0}{1/2} \cdot 2 c t_0$$

* what's the distinction bet. this & the
Hubble distance? $l_{Hab} = c t_0 = c \frac{1}{H_0}$

$$a = (2\sum_{i=0}^{n_2} H_0 t)^{1/2}$$

4

exp. rate
(velocity)

$$\frac{da}{dt} = (\sum_{i=0}^{n_2} H_0 t)^{-1/2} 2\sum_{i=0}^{n_2} H_0$$

$$\frac{d \text{rate}}{dt} = \text{acc. : } \frac{d^2 a}{dt^2} = -\frac{1}{2} (-)^{-3/2} (2\sum_{i=0}^{n_2} H_0)^2$$

exp. slows w/ time
(due to gravity)

{some other cases}

p. 557 "With complete knowledge of $a(t)$, the time corresponding to any scale factor can be computed."

... e.g. eq. (24.33) ~~\propto~~ ³³ ~~t_0~~ ~~\propto~~ ^{t_0} corresponds to a time:

$$\cancel{\propto} \frac{1}{H_0}$$

$$t_0 = 0.964 H_0^{-1} = 13.5 \text{ Gyr}$$

recall: empty $t_0 = 1/H_0$

$$\sum_m = 1, \sum_n = 0, t_0 = \frac{2}{3} \cdot \frac{1}{H_0}$$

$$\sum_m = 0, \sum_n = 1, t_0 = \infty$$

$$\Rightarrow \sum_m = 0.3, \sum_n = 0$$

is in between