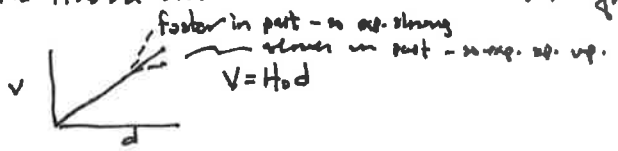
$$F_{exp} = \frac{L}{4\pi r^2 (1+z)^2}$$; r is the proper distance to the galaxy

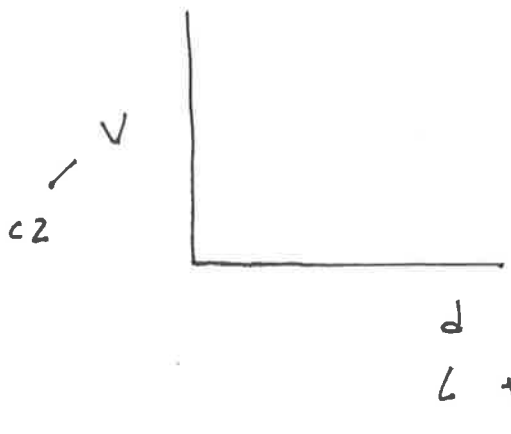
└ diff. from inv sq. law

— redshift affects energy
— each successive photon is emitted from farther away

basic Hubble law:



[m – M is like $\frac{L}{F_{exp}}$]



⇝ really

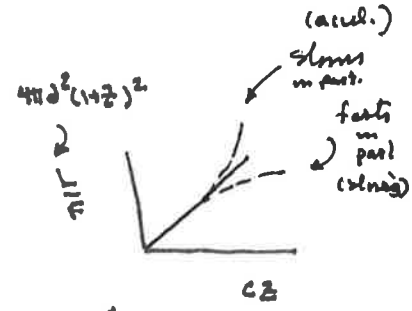


Fig. 24.4

↳ qualitatively like Fig 24.5

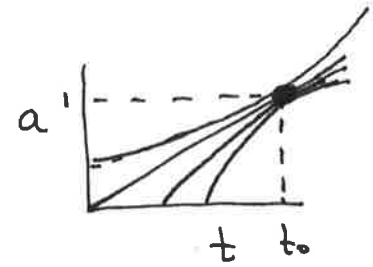
How do we quantify this & use real data to choose among models?

$$l_p = c\int_{t_e}^{t_0} \frac{dt}{a}$$

- so, given an observed z
  $1+z = 1/a$ → find a
- given a model universe (a(t))
  we can find the $t = t_e$
  at which $a = \frac{1}{1+z}$
- Then we can solve l_p eqn.
  given that t_e & a(t)

$a = a_0 (t/t_0)^{2/3}$ for Newtonian, critical universe

$\uparrow 1$



if $z = 3$ then $a = \frac{1}{1+z} = \frac{1}{4}$

$\frac{1}{4} = (t_e/t_0)^{2/3}$

$t_e/t_0 = (\frac{1}{4})^{3/2} = \frac{1}{8}$

so $$l_p = c\int_{t_e}^{t_0} \frac{dt}{a} = c\int_{t_0/8}^{t_0} \frac{dt}{(t/t_0)^{2/3}}$$

$$= c t_0^{2/3} \int_{t_0/8}^{t_0} t^{-2/3} dt = c t_0^{2/3} \left[3t^{1/3} \Big|_{t_0/8}^{t_0} \right]$$

$$= c t_0^{2/3} \left[3t_0^{1/3} - 3\left(\frac{t_0}{8}\right)^{1/3} \right]$$

$$= 3 c t_0 \left[1 - \frac{1}{2}\right]$$

$$= \frac{3}{2} c t_0$$

$$= c/H_0$$

? for an empty universe: $a = a_0 (t/t_0)$

if $z = 3$ then $a = \frac{1}{1+z} = \frac{1}{4}$

$\frac{1}{4} = t_e/t_0 \rightarrow t_e = \frac{t_0}{4}$

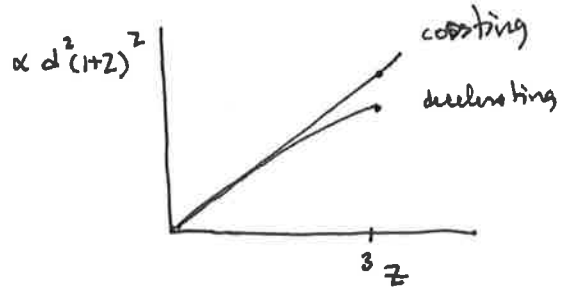
$$l_p = c\int_{t_e}^{t_0} \frac{dt}{a} = c\int_{t_0/4}^{t_0} \frac{dt}{t/t_0}$$

$$= c t_0 \int_{t_0/4}^{t_0} t^{-1} dt$$

$$= c t_0 \left(\ln t \Big|_{t_0/4}^{t_0} \right)$$

$$= c t_0 (\ln t_0 - \ln t_0/4) = c t_0 (\cancel{\ln t_0} - \cancel{\ln t_0} + \ln 4)$$

$$= c t_0 \, 1.39 = c \, 1.39/H_0$$

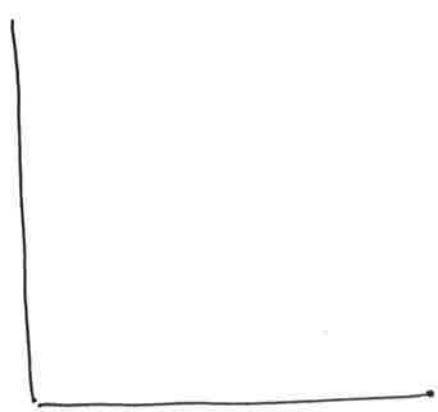


$z = 3$ $\quad l_{p\,\Omega=0} = 1.39\,ct_0 = \frac{1.39c}{H_0}$

$z = 3$ $\quad l_{p\,\Omega_m=1} = \frac{c}{H_0}$

* $\Omega_m \rightarrow$ slows down

$\Omega_\Lambda \rightarrow$ speeds up.

for hybrid ($\Omega_m \neq 0, \Omega_\Lambda \neq 0$) models, $\frac{\Omega_\Lambda}{\Omega_m}$ give roughly the same expansion history

(but v. early times — slow down-to-speed up transition z — timing is affected)