

1st time
review of
CMB

$$F = ma$$

$$F = F_g = - \frac{GMm}{r^2}$$

only gravity acts on
galaxy (symmetry: ignore other)

(see slides from Nov. 1st)

$$\ddot{r} = \frac{d^2 r}{dt^2} = a = - \frac{GM}{r^2}$$

integrate both sides:

$$\frac{1}{2} \left(\frac{dr}{dt} \right)^2 = \frac{GM}{r} + K$$

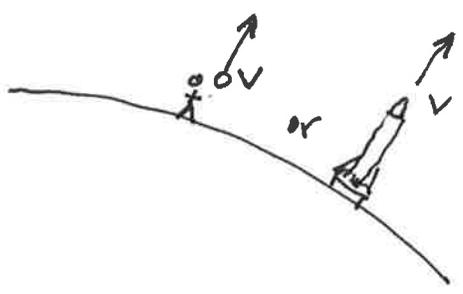
* Differentiate to ~~check~~ check — recover original eqn.

energy conservation

$$\overset{KE}{\frac{1}{2}mv^2} + \overset{PE_g}{\frac{GMm}{r}} = \text{const.}$$

analogy

↑
< 0, 0, > 0



Now: change of variables:

$$r(t=now) = r_0$$

$$r(t) = a(t) r_0$$

* $a(t)$ describes the history & future of the universe; age, too
 (calculate soon) $a(t=t_{bb}) = 0$

* Q: What is the value of $a(t)$ today?

$$M_{inside} = \text{Vol.} \times \text{density} = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi r_0^3 a^3$$

$$\frac{1}{2} \dot{a} r_0^2 = G \frac{\frac{4}{3}\pi r_0^3 a^3 \rho}{a r_0} + \frac{K}{a}$$

$$\dot{a}^2 = \frac{8\pi G \rho}{3} a^2 + \frac{2K}{r_0^2 a}$$

divide by a^2

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho + \frac{2K}{r_0^2 a^3}$$

* fncs of time?

now, since Hubble Law: $v = H_0 d$
 \uparrow here $d = r$

$$H_0 = \frac{v}{r} \quad \text{but } v \equiv \frac{dr}{dt} \equiv \dot{r}$$

$$\Rightarrow H_0 = \frac{\dot{r}}{r} \quad ; \quad r = a r_0 \quad ; \quad \dot{r} = r_0 \dot{a}$$

$$\Rightarrow H_0 = \frac{\dot{a}}{a} \quad \text{but "0" subscript means "evaluated now"} \\ \text{more generally } H = \frac{\dot{a}(t)}{a(t)}$$

thus —

$$H_0^2 = \frac{8\pi G}{3} \rho + \frac{2k}{r_0^2 a^2}$$

\uparrow expansion \uparrow gravity \uparrow

note: $a > 0$ $r_0 > 0 = \text{const}$
 so ~~the~~ ~~the~~ the
 sign of this term
 is fully determined by k

this is the Friedmann Eqn.

classically derived BUT same result from GR

however, in GR, 2nd term on r.h.s.

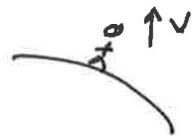
represents the curvature of spacetime.

For now, let's focus on dynamics:

will the expansion continue forever or halt (i. reverse)?

$k > 0 \rightarrow$ expand forever

$k < 0 \rightarrow$ recollapse



(later: $k > 0 \rightarrow$ hyperbolic

$k < 0 \rightarrow$ spherical

... $k = 0 \rightarrow$ flat/Euclidean

$$KE = -PE + \text{const}$$

$$KE + PE = \text{const.}$$

(neg.)
grav. $L > 0$

$$KE > |PE|$$

projectile
never going

Q: dynamically, what does $k=0$ corresp. to?

\hookrightarrow asymptotic expansion

(like $v_{\text{proj}} = v_{\text{esc}}$ exactly)

* this is $k=0$ case is the "critical" case:

[Note: only one, unique $k=0$ value,
whereas infinite $k < 0, k > 0$ values]

Let's examine the "critical universe" —

$$H^2 = \frac{8\pi G}{3} \rho + \frac{2k}{r_0^2 a^2}$$

↳ set to 0

$$H^2 = \frac{8\pi G}{3} \rho_{\text{crit}}$$

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}$$

* Q: is it a func. of time?

its current value:

$$\rho_{\text{crit}_0} = \frac{3H_0^2}{8\pi G}$$

$$= 9.2 \times 10^{-27} \text{ kg/m}^3 = 1.4 \times 10^{11} M_{\odot} / \text{Mpc}^3$$

* What is the actual density?

↳ count up galaxies

define $\Omega \equiv \frac{\rho_{\text{actual}}}{\rho_{\text{critical}}}$

* talk about time dependence:

* Q:

$$\rho_{\text{crit}}(t) \propto H(t)^2$$

$$\rho_{\text{actual}}(t) = \rho_0 a(t)^{-3}$$

↳ in terms of ρ_0 & a ?