



$$r(t) = r_0 a(t)$$

$$\text{where } a(t=\text{now}) = a_0 \equiv 1$$

note: homogeneity & isotropy

imply $a \neq f(r)$

a only $f(t)$

$$\text{P.SD} \quad v \equiv \dot{r} = \dot{a}r_0 = \frac{\dot{a}}{a} r = H_0 d \quad \checkmark$$

$$\hookrightarrow H_0 = \frac{\dot{a}}{a}$$

what is $a(t)$?

numerical:

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3} p + \frac{2k}{r_0^2 a^2}$$

\uparrow
like
 κE

\uparrow
like
 $g_{\mu\nu} P E$

\uparrow
Newton:
cont. (Newton - relativity - analogy - fate)
 \uparrow
EINSTEIN: curvature

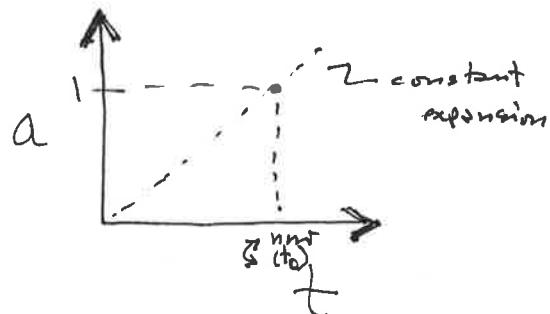
$\kappa > 0$ open - expands forever
 $\kappa < 0$ closed - stops, collapses (big)
 $\kappa = 0$ critical

$\kappa = -1$ very hyperbolic $\kappa = 0$ flat
 $\kappa = +1$ spherical

$$\frac{\dot{a}}{a} = H$$

$$H^2 = \frac{8\pi G}{3} p + \frac{2k}{r_0^2 a^2}$$

e.g. constant expansion



let's examine critical case $K=0$ ($\therefore K=0$, flat - Euclidean)

$$H^2 = \frac{8\pi G}{3} \rho_{\text{crit}}$$

$$\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G}$$

$$\rho_{\text{crit}_0} = \frac{3H_0^2}{8\pi G}$$

$$H_0 = 70 \text{ km/s/Mpc}$$

* will measure the actual density, ρ ;

compare it to ρ_{crit}

$$\rho_{\text{crit}_0} = \del{4.2} 9.2 \times 10^{-27} \text{ kg m}^{-3}$$

$$\rho_{\text{crit}_0} = 1.4 \times 10^{11} M_\odot \text{ Mpc}^{-3}$$

? MW? scale?
actual masses

$$\rho/\rho_{\text{crit}} = \sum \text{ def } n$$

ask students

$$\text{so, } \Omega = 1 \Rightarrow \text{critical case, } K=0 \quad \rightarrow$$

$$\Omega < 1 \Rightarrow K > 0 \rightarrow \text{open}$$

$$\Omega > 1 \Rightarrow K < 0 \rightarrow \text{closed}$$

* measured in terms of strength of gravity

* Q: what is the time dependence of ρ_{crit} ? ρ ?

$$\rho_{\text{crit}} \propto H^2$$

$$\rho \propto \rho_0 a(t)^{-3}$$

so in critical universe
 $H = \sqrt{\frac{8\pi G}{3} \rho_0} a^{-3/2}$

observational census of $\Omega \Rightarrow \Omega$

- mass-light ratio ...
- grav. mass \rightarrow dark matter
- use of grav. lensing

1990's \rightarrow

observ. result:

$$\Omega_{\text{baryon}} \lesssim 0.05$$

\nearrow
functionality light smoothing

$$\Omega_{\text{grav}} \lesssim 0.3$$



next time?

(or soon...)

frustrate: $\Omega = 1 \rightarrow$ critical universe: flat, exp. forever but w/ no margin

$\Omega < 1 \rightarrow$ hyperbolic, exp. forever w/ rel. to space (open)

$\Omega > 1 \rightarrow$ spherical, exp-stops, recollapses (closed)

measure:

up to 1990's guest: H_0, Ω ($\therefore \Omega$ implies $g \propto a^{-2}$ which, in principle could be measured directly)
 also, curvature itself could
 (sign of K) be measured.

back to the CMB

recall: $\rho \propto T^4$

$\left\{ \begin{array}{l} \\ \text{energy density} \end{array} \right.$

but energy density - of an ensemble of particles -
should scale like the density of those particles
times the energy of each particle

recall: $\rho(t) \propto a^{-3}$ for baryons
but also for photons

however energy/particle of photons

scales as $h\nu \propto a^{-1}$ redshift

$$\frac{\lambda}{\lambda_0} = \frac{a}{a_0} = \frac{a}{t} = \alpha(t)$$

* source of cosmological redshift

$$T(t) \propto \rho(t) \cdot h\nu(t) \propto a^{-3} \cdot a^{-1}$$

$$T_{\text{CMB}}(t) \propto a^{-4}$$

but also $T_{\text{CMB}}(t) \propto T^4$ so $T \propto a^{-1}$

yet! $\rightarrow \lambda_{\text{peak}} T = \text{const.}$

$$\lambda_{\text{peak}} \propto 1/T$$

but redshift $\lambda \propto a \Rightarrow T \propto 1/a$ RED.

