

Robertson Walker metric

$$d\ell^2 = -c^2 dt^2 + a^2(t) d\tau^2$$

↑ comoving
coord.
r
so gal. distance
is τ_0 always

"regular" (Minkowski)
metric from SR

But modified
to include the
expansion of the Univ.

via scale factor $a(t)$

time is cosmic time -

measured by observers at rest,
seeing the Univ. expand uniformly
away from himself.

proper distance = length of geodesic^{shortest distance - best path}
at a fixed time, t (so - a measuring stick
would measure this value)

proper distance in a flat Univ. can be found from the RW/M

"fixed time, t " $\Rightarrow dt = 0$

so $d\ell = a(t) d\tau$

note: ℓ is a func. of t
 $\ell(t)$ [though we
often want to
evaluate it now,
 $t = t_0$]

integrate to find ℓ

$$\int d\ell = \int a(t) d\tau$$

$$\ell(t) = a \int_0^{\tau} d\tau = a(t) \tau$$

since $a(t_0) = 1$

$$\ell(t_0) = \tau = \tau_0$$