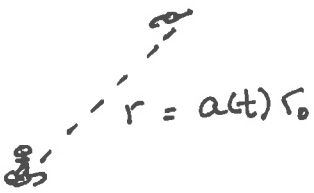
* Thanksg. Wed. ?

* outline rest of classes...



* $r(t)$ IS the proper distance to one (or a representative set) of galaxies *

Quest is to know $a(t)$ for all $t < t_0$
including the age of the Universe $t_0 - t_{bb}$
AND to extrapolate into the future $t > t_0$

dynamics requires knowing force-like terms
gravity
curvature
cosm. const.

just description of the past though only requires measurements/knowledge of $a(t, t<t_0)$

we ask, what can we measure that provides information about

$$a(t, t<t_0) \quad ?$$

to measure distances, we need a <u>metric</u>

plain old Sp. Rel. metric $ds^2 = dt^2 - dr^2$

different sign convention $ds^2 = -dt^2 + dr^2$

plus, they put the speed of light in explicitly, rather than use units where $c = 1$

so

$$ds^2 = -c^2 dt^2 + dr^2$$

is the (spherically symm.) metric — plainest spec. rel. version: "Minkowski metric"

two applications:

i) to measure the <u>length</u> of something

set $dt = 0$ (since you measure the locations of each end of the object at the same time)

ii) $ds = 0$ for photons (because $v = c$ for them), so to calculate the time it takes a photon to travel a certain distance, set $ds = 0$

$$c\,dt = dr \quad \rightarrow \quad dt = \frac{dr}{c} \;;$$

integrate both sides

We incorporate the expansion of the Universe into the metric:

$$ds^2 = -c^2 dt^2 + a(t)^2 dr^2$$

↑ scale factor

Robertson-Walker metric

* we can play the same tricks:

i) length: $dt = 0$ &

$$dl_p = a\,dr$$ (eqn. 23.54, p 544)

ii) photon's travel time: $ds = 0$

$$c\,dt = a(t)\,dr$$ (eqn. 23.56 p. 545)

Now we can relate the things we'd like to know – l_p & $a(t)$ – to things we can measure

take $c\,dt = a(t)\,dr$ & separate t- & r- dep. terms

$$\frac{c\,dt}{a(t)} = dr$$

integrate along a photon's path:

$$c\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^r dr = r$$

↳ this is l_p

$r = 0$ (from photon's P.O.V.)
$t = t_e$ (time photon is emitted)

$r = r = r_0 a$ (distance the galaxy is away from us – & we from them – NOW)
$t = t_0$

$$l_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

↑ proper distance of galaxy now

trivially, in a static universe ($a = 1$ at all t)

$$l_p = c(t_0 - t_e)$$

But in an expanding Universe $a < 1$ in the past

so $$\int_{t_e}^{t_0} \frac{dt}{a} > \int_{t_e}^{t_0} dt$$

$$\therefore \quad l_p > c(t_0 - t_e)$$

* proper distances get bigger in an expanding Universe

* "$l_p(t_0)$ isn't something we can measure, it's something we can compute if we know $a(t)$"

* went over prob. 23.4 $a(t) = \left(1 - \frac{3}{2}H_0(t_0 - t)\right)^{2/3}$

→ Mathematica plot

aside: $H(t, r)$ as a way to verify universal time

<u>But</u> we get information from redshift

t_e is related to λ_e ← wavelength of light when it was emitted (lab wavelength)

$$\frac{\lambda_e}{a(t_e)} = \frac{\lambda_0}{a(t_0)} = \lambda_0$$

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e}$$ note λ_e is lab-wavelength.

$$1 + z = \frac{\lambda_0}{\lambda_e} = \frac{1}{a(t_e)}$$

(on fri:)

aside: $T_{CMB} = T_0 / a(t_e)$ [hotter in the past]

{ could also eg 23.5 prove BB redshifted is a BB w/ $T = T_0/1+z$

§23.5 Friedmann equation from G.R. → dynamics

Newton: $$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3}\rho + \frac{2K}{r_0^2 a^2}$$

↑ Newtonian: energy

$K>0$ open
$K=0$ critical
$K<0$ closed

Einstein: $$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3c^2}\upsilon - \frac{Kc^2}{r_0^2 a^2} + \frac{\Lambda}{3}$$

↖ cosm. const.

mass + energy density

$\upsilon = \upsilon_m + \upsilon_r$

↑ radiation

Einstein: curvature

$K=-1$ hyperbolic
$K=0$ flat
$K=+1$ spherical

energy – mass

$E = mc^2$

energy density – mass density

$\upsilon = \rho c^2$ so $\upsilon_{crit} = \rho_{crit} c^2$

can also express energy density in Λ → υ_Λ

so $$\Omega \equiv \rho/\rho_{crit}$$

↳ summarize on board Fri 1st

$$\Omega_m \equiv \rho_{matter}/\rho_{crit}$$

$$\Omega_r \equiv \upsilon_{rad}/\rho_{crit}c^2$$

$$\Omega_\Lambda \equiv \upsilon_\Lambda/\rho_{crit}c^2$$

$$\Omega = \Omega_m + \Omega_r + \Omega_\Lambda$$

$\Omega > 1$ spherical
$\Omega = 1$ critical/flat
$\Omega < 1$ hyperbolic