

Today - Using F. Eqs. to examine models of
The Universe ($a(t)$)

↳ ultimately - apply obs. constraints to
figure out what $a(t)$ describes
The Universe we actually live in

(soon) \rightarrow how to relate $a(t)$ to the Hubble
Diagram (brightness (or distance) vs. redshift)

Topics —

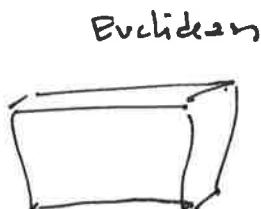
- CMB anisotropies as constraints on Geometry
- Hubble parameter @ high redshift \rightarrow accelerating universe ($\Lambda > 0$)

Three geometries (only) that are consistent
with the Cosmological principle



Spherical

$$K = +1$$



Euclidean

$$K = 0$$



Hyperbolic

(saddle-shaped)

$$K = -1$$

note: 2-D analogies

Different (than Euclid's) geometries — e.g. rules about:

— triangles —

— parallel lines — \neq side

— angular sizes

* spherical & hyperbolic have parameters

- r_0 — that describe the ^{length} scale
of their curvature

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} U - \frac{Kc^2}{r_{c,0}^2} \cdot \frac{1}{a^2} + \frac{\Lambda}{3}$$

↑
 geometry
 term

⇒ observations indicate Universe is close to flat ($K=0$) ($\sum L = 1$)

basic obs — pretty close

detailed — much closer

let's look at basic constraints

recall: Hubble distance : $c/H_0 \approx 4.3$ Gpc

we can show that if $K \neq 0$, then

$$r_{c,0} > c/H_0$$

angle - size - distance

$$\theta = \frac{D}{d}$$

$$d \longrightarrow$$

$$\theta = D/d$$

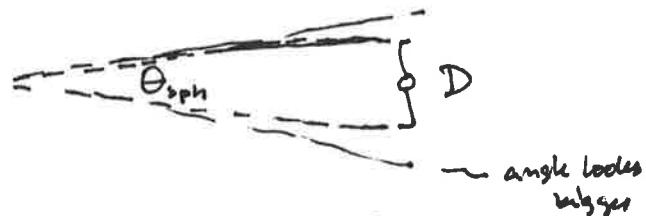
\uparrow radians $\uparrow D, d$ in same units

some numbers: $\theta = 1'' \sim$ smallest angle measurable

$1''_{\text{arcsec}} = \frac{1}{200,000} \text{ radian}$ w/ a ground-based optical telescope

$$D \lesssim 100 \text{ kpc} \quad (0.1 \text{ Mpc} = 10^{-4} \text{ Gpc})$$

\therefore galaxies can be resolved
out to Hubble distance

euclidean $K=0$ spherical $K=+1$
religion ~~KG~~hyperbolic $K=-1$

Extreme : galaxy near S.P. - blocks entire hemisphere ($\theta \sim 180^\circ$!)

$$\alpha_{\text{Euc}} = \frac{D}{d}$$

$$\alpha_{\text{sph}} = \frac{D}{r_{c,0} \sin(\pi d / r_{c,0})}$$

$$\alpha_{\text{hyp}} = \frac{D}{r_{c,0} \sinh(\pi d / r_{c,0})} \sim \frac{2D}{r_{c,0}} e^{-\pi d / r_{c,0}}$$

* plot these