Assignment 1
Due in class on Tuesday, January 22

Answer the numbered questions. Show your work, provide explanations, use sketches, indicate units.

There are quite a few ways that exoplanets might conceivably be detected. In this set of problems, we'll look at four ways: direct imaging, astrometric (spatial displacement due to orbital motion), spectroscopic (or radial velocity, Doppler shift due to orbital motion), and transit (periodic dimming of starlight due to planet being between its host star and the Earth).

Along the way, we'll encounter a few broadly useful concepts: angular measure, the inverse square law of light, the Doppler shift, Kepler's third law. I expect that the various students in the class will have had different levels of prior exposure to these concepts. Since there are particularly useful ways I'd like you to think about these concepts, we will go over them explicitly. We'll start with this set of related problems:

> *Taking the solar system to consist only of the Sun and Jupiter (not a bad approximation, if you consider mass or brightness), what measurable signal of Jupiter's presence could be detected directly, astrometrically, spectroscopically, or via the transit method by a "nearby" observer?*

You should use either of the sets of constants I've linked from the main class webpage (note a new set of links at the extreme lower right) or from some other source (but then cite, explicitly). Note that one link has them in the cgs system. Professional astrophysicists use cgs. Educators, students, and physicists use mks (also known as SI). Take your pick, I say! Just be clear about what you're doing.

We'll talk about direct imaging of exoplanets in class on Tuesday. It is extremely difficult[1]. Planets are dim. Stars are bright. There are special difficulties associated with measuring the brightness of a dim thing very close to a bright thing. Understanding this involves thinking about angular separation.

When you look at two objects in the sky (two stars in Orion's belt, say) and perceive them to be close together, what you're assessing is their *angular separation*. The two stars may be significantly different distances away from us and thus from each other, but projected onto the plane of the sky, they are close together in angle[2].

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[1] There is, however, no doubt that it will one day be done routinely.
[2] From our point of view, that is. Alien observers in a different part of the galaxy would see them as having a different angular separation.

1. If you were observing the Solar System from a distance of 3.26 light years (roughly the typical inter-star distance in our region of the Galaxy), what would the angular separation of the Sun and Jupiter be? Please give your answer in radians and in arc seconds. You may take Jupiter's orbit to be circular.

In order to solve this, please sketch an observer, the Sun, and Jupiter (the first as a stick figure or an eye and the later two just as points). Label the relevant angle, the distance between Jupiter and the Sun, and the distance from the observer to a point half-way between Jupiter and the Sun. Bisect the angle and now you've got two identical right triangles. You should be able to derive an expression relating the angle, separation, and distance.

Feel free to use the small angle approximation (sine and tangent of an angle are both equal to the value of the angle itself (in radians) for small angles) if you'd like. I prefer it, personally.

2. How many arc seconds are there in a radian (to six significant figures, please)?

3. While you're at it, calculate the apparent angular diameter of the Sun from your point of view, 3.26 light years away. That is, calculate the angular separation of the left edge of the Sun from its right edge.

Please note that turbulence in the Earth's atmosphere limits spatial resolution to about one arc second. That is all objects that appear smaller than this are "smeared out" to a size of about one arc second, and their true size cannot be measured. Stars all appear to be "point sources" – spatially unresolved, we say – because even the nearest ones are so very far away. Similarly, if two bright objects are closer together than about one arc second, we will only see them as one blob of light; we won't be able to tell that there are two objects there; they will be *unresolved*.

4. So, if one arc second is the limit, could you see the Sun and Jupiter as two distinct entities and thus identify Jupiter as an exoplanets via direct imaging? Of course, the extreme dimness of Jupiter compared to the Sun will be a factor, too.

5. Can you make a rough estimate of the relative brightness of these two objects? Use the Jupiter-Sun distance and Jupiter's cross-sectional area to figure out what fraction of the Sun's light is intercepted by Jupiter (you should consider an imaginary sphere centered on the Sun, with a radius equal to the Jupiter-Sun distance). This fraction should approximate the ratio of Jupiter's brightness to the Sun's. What other factors can you think of that would affect Jupiter's relative brightness?

Interestingly, although direct imaging of a Jupiter-like planet orbiting a Sun-like star at the same distance Jupiter does is not (currently) possible, due to the extreme brightness contrast, the light of an exoplanet closer-in to its star can in some cases be detected in the combined star+exoplanet's light.

Next: astrometry. Astrometry is the precise measurement of stellar positions (including how they change, minutely, over time). The Sproul Observatory on campus (big green dome near Tarble-Clothier) did a lot of fundamental stellar astrometry in the middle of the 20th Century.

We just calculated the Jupiter-Sun angular separation, but saw that the extreme brightness contrast means that we couldn't measure Jupiter's presence (much less its position or changing position as it orbits the Sun) so what else is there to consider? Jupiter does not orbit a stationary Sun, but rather they both orbit their common center of mass (as you saw in the binary star applet). So, the Sun moves in a (nearly) circular orbit around the center of mass due to the gravitational pull of Jupiter. Astrometric detection of Jupiter from your vantage point 3.26 light years away would involve measuring the Sun's apparent motion on the plane of the sky as it moves in its orbit.

If the distances of the Sun and Jupiter from their common center of mass are given by a_S and a_J, where $a_S + a_J = a$, the semimajor axis of their orbit (and for circular orbits, which we're assuming here, the separation of the two objects), then

$$a_J M_J = a_S M_S.$$

6. How far from the Sun's center is the Jupiter-Sun center of mass? Give your answer in terms of the Sun's radius (how many solar radii?).

7. What is the angular size of the Sun's orbit from your vantage point 3.26 light years away? Express your answer in arc seconds. How does this angle compare to the 1 arc second atmospheric seeing limit?

What about trying to detect the Sun's motion via the Doppler shift? (Don't lose track of the fact that yes, Jupiter is moving much faster than the Sun – because its orbit is bigger – but we can't measure its Doppler shift because we can't collect enough undiluted light from it.) In the radial velocity method, the motion of the star around the star+exoplanet center of mass is measured via the time-varying Doppler shift, which goes through one full cycle over the course of one orbital period.

8. Calculate the Sun's orbital velocity given a_S computed above and the orbital period of Jupiter (which must be equal to the orbital period of the Sun in a hypothetical system with only those two bodies). Express that velocity in m/s, miles per hour, and also as a fraction of the speed of light. Note that the Doppler shift is given by

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$

where v_r is the radial velocity, or the component of the velocity along our line of sight (perpendicular to the plane of the sky), c is the speed of light, and the Greek letter lambda, λ, is the wavelength of a particular spectral line. The change in wavelength, $\Delta\lambda$, is the difference between the wavelength of that spectral line in your measured spectrum and what it would be if it were emitted from a stationary object.

Amazingly, the small velocity you calculated above can be measured via spectroscopy of starlight. And the period of the radial velocity variation gives the orbital period of the planet, while the radial velocity amplitude is proportional to the mass of the planet. So, if you could measure that small solar "reflex motion" via the Doppler shift, you could, in principle, weigh Jupiter. Additionally, knowing the orbital period and the mass of the star (which can be figured out pretty well via traditional astrophysical methods) Kepler's third law tells us the semi-major axis of Jupiter's orbit. This radial velocity method was the means of discovery of a large majority of the first few hundred exoplanets found. And it will always be the best way to characterize an exoplanet's mass.

Think about how the *orientation* of the Jupiter-Sun orbit with respect to you, the observer, affects the radial velocity (and hence Doppler shift) that you'll measure.

Finally, the transit method.

9. If Jupiter passed in front of the Sun from your vantage point, by what fraction would the Sun's brightness dim? You can take both the Sun and Jupiter to be spherical, and thus to have circular cross sections. The Sun can assumed to be uniformly bright across its circular disc while Jupiter can be assumed to be absolutely dark and opaque.