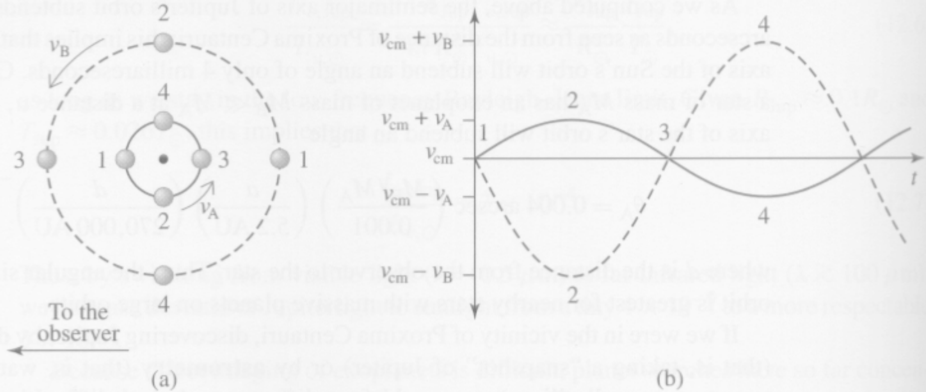

FIGURE 12.5 (a) A star (solid orbit) and a planet (dashed orbit) moving about their center of mass. The observer is in the orbital plane, far to the left. The numbers indicate the locations of the star and planet at specific times. (b) The radial velocity of the star (solid curve) and the planet (dashed curve) relative to the observer. The numbered positions correspond to the locations indicated in part (a).

Thus, the Sun's orbital speed is a modest $13\,\mathrm{m\,s^{-1}}$, or about 29 mph.[2] With current techniques, the radial velocity of relatively bright stars can be determined to within $\sim 1\,\mathrm{m\,s^{-1}}$; thus, it is technologically feasible to detect Jupiter-like exoplanets from their effect on the radial velocity of Sun-like stars.

Suppose that we are looking at a system that consists of a star and a single planet. The center of mass of the system will be moving with some radial velocity v_{cm}. In addition, if the star and planet are on circular orbits, and we are in the orbital plane of the system (as shown in Figure 12.5a), the radial velocity of the star will show sinusoidal variations. The sinusoidal curve (as shown in Figure 12.5b) will have an amplitude v_A and a period P equal to the orbital period of the planet. In general, though, we will not be in the orbital plane of the system; we will be observing the orbital plane at an inclination i, as illustrated in Figure 12.6. Thus, the measured amplitude of a radial velocity curve is only equal to v_A if we are observing the orbital plane edge-on ($i = 90°$); at all other orbital inclinations, the amplitude is $v_A \sin i < v_A$.

From the velocity curve of a star with an unseen companion, we can determine the orbital period P and the value of $v_A \sin i$. Let's see how we can use these measured quantities to deduce the properties of the perturbing exoplanet. Kepler's third law, as modified by Newton (equation 3.53), tells us that

$$(a_A + a_B)^3 = \frac{G(M_A + M_B)P^2}{4\pi^2}. \tag{12.15}$$

[2] For comparison, this also happens to be the average air speed of an unladen European swallow (Park et al., 2001, *Journal of Experimental Biology*, vol. 204, p. 2741).

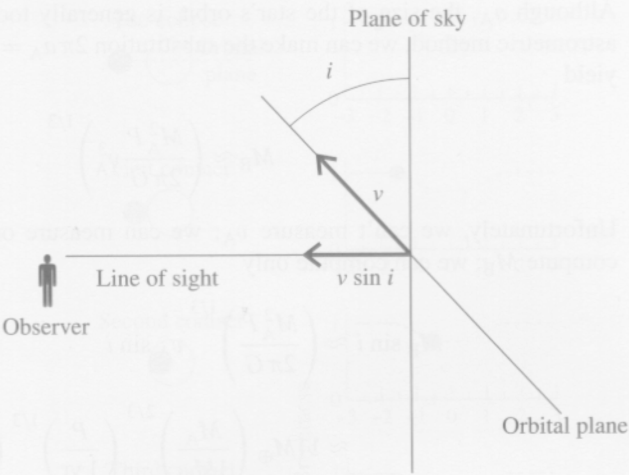

FIGURE 12.6 The maximum radial velocity v_r that we measure is the projection of the orbital speed v onto the line of sight: $v_r = v \sin i$.

In the case that $M_A \gg M_B$, and hence $a_A \ll a_B$, we can simplify this to

$$a_B^3 \approx \frac{G M_A P^2}{4\pi^2}. \qquad (12.16)$$

The mass M_A of the star, as we see in Chapter 13, can usually be deduced from its spectrum. For instance, stars with spectra similar to the Sun's have masses close to a solar mass. Knowledge of the star's mass enables us to estimate the size of the planet's orbit:

$$a_B \approx 1\,\mathrm{AU} \left(\frac{M_A}{1 M_\odot}\right)^{1/3} \left(\frac{P}{1\,\mathrm{yr}}\right)^{2/3}. \qquad (12.17)$$

In addition, the amplitude of the velocity curve, $v_A \sin i$, gives us some knowledge of the mass of the planet. Using the relation $a_B = (M_A/M_B) a_A$, we start by rewriting equation (12.16) in the form

$$\frac{M_A^3}{M_B^3} a_A^3 \approx \frac{G M_A P^2}{4\pi^2}. \qquad (12.18)$$

Solving for M_B, the planet's mass, we find

$$M_B \approx \left(\frac{4\pi^2 M_A^2}{G P^2} a_A^3\right)^{1/3}. \qquad (12.19)$$

Although $a_{\rm A}$, the size of the star's orbit, is generally too tiny to be measured by the astrometric method, we can make the substitution $2\pi a_{\rm A} = P v_{\rm A}$ into equation (12.19) to yield

$$M_{\rm B} \approx \left(\frac{M_{\rm A}^2 P}{2\pi G} v_{\rm A}^3 \right)^{1/3} . \tag{12.20}$$

Unfortunately, we can't measure $v_{\rm A}$; we can measure only $v_{\rm A} \sin i$. Thus, we can't compute $M_{\rm B}$; we can compute only

$$M_{\rm B} \sin i \approx \left(\frac{M_{\rm A}^2 P}{2\pi G} \right)^{1/3} v_{\rm A} \sin i$$

$$\approx 11 M_{\oplus} \left(\frac{M_{\rm A}}{1 M_{\odot}} \right)^{2/3} \left(\frac{P}{1\,{\rm yr}} \right)^{1/3} \left(\frac{v_{\rm A} \sin i}{1\,{\rm m\,s^{-1}}} \right) . \tag{12.21}$$

In the above equation, $M_{\oplus} = 6.0 \times 10^{24}$ kg is the Earth's mass; it takes 318 Earths to equal the mass of one Jupiter.

If an exoplanet is to be detected by the radial velocity method, it must produce $v_{\rm A} \sin i$ large enough to measure. Thus, exoplanets whose orbits are more nearly edge-on ($\sin i \sim 1$) are more readily detected. In addition, since $v_{\rm A} \propto M_{\rm B} M_{\rm A}^{-2/3} P^{-1/3}$ (from equation 12.21), the radial velocity method favors the detection of massive, short-period planets around relatively low-mass stars. In addition, measuring P accurately requires observing the star for a time $\sim P$ or longer; this adds to the difficulty of detecting long-period exoplanets.

The detection of exoplanets by the **transit** method requires a different set of observations, and favors the detection of exoplanets with somewhat different properties than those found by the radial velocity method. In the language of astronomers, a "transit" is the passage of a planet between its parent star and the observer. Within the solar system, for instance, observers on Earth can occasionally see transits of Mercury and Venus across the Sun. During such a transit, the inferior planet appears as a small, dark circle against the bright circle of the Sun, as shown in Figure 7.5. When an exoplanet makes a transit, neither the star nor the exoplanet can be resolved in angle as seen from the Earth. However, the transiting planet can be detected indirectly since it blocks part of the star's flux. If the cross-section of the planet is $\pi R_{\rm B}^2$ and the cross-section of the star is $\pi R_{\rm A}^2$, then when the planet lies directly between the star and an observer, the star's measured flux F drops by a fractional amount

$$\frac{\delta F}{F} = \frac{\pi R_{\rm B}^2}{\pi R_{\rm A}^2} = \left(\frac{R_{\rm B}}{R_{\rm A}} \right)^2 . \tag{12.22}$$

If a distant observer saw Jupiter transit across the Sun, for instance, the Sun's flux would drop by a fraction

$$\frac{\delta F}{F} = \left(\frac{R_{\rm Jup}}{R_{\odot}} \right)^2 = \left(\frac{69{,}900\ {\rm km}}{696{,}000\ {\rm km}} \right)^2 = 0.010, \tag{12.23}$$

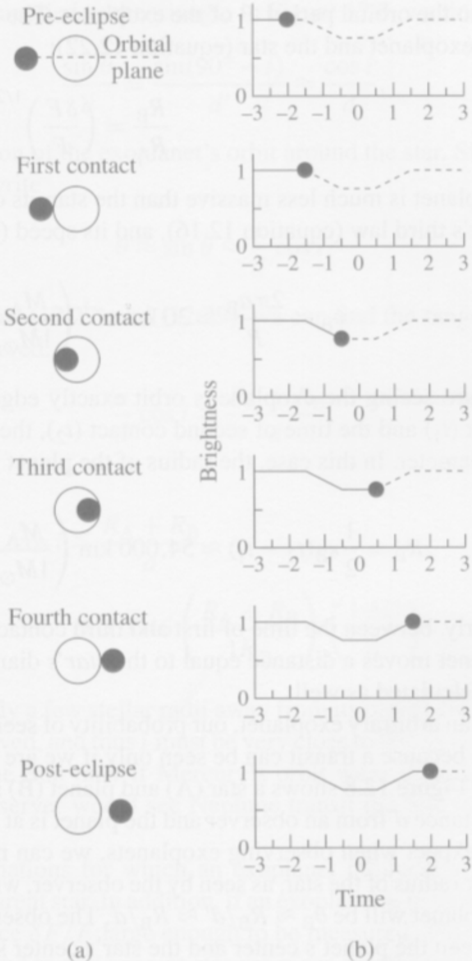

FIGURE 12.7 (a) Events when a planet transits a star. (b) The light curve (measured flux as a function of time) during the transit, with a marker indicating the flux during the corresponding event in part (a).

since Jupiter would cover 1% of the Sun's area.[3]

The light curve of a star during a transit is shown in more detail in Figure 12.7.

From the light curve, we can measure the drop in flux at mid-transit, $\delta F/F$; we can also measure the times of first, second, third, and fourth contact, as defined in the figure. If we wait patiently, we can also measure the time between successive transits; this is

[3] This simple calculation assumes that the Sun's surface brightness is uniform. The effects of limb darkening (described in Section 7.1) slightly complicate matters.

equal to the orbital period P of the exoplanet. The drop in flux tells us the relative sizes of the exoplanet and the star (equation 12.22):

$$\frac{R_B}{R_A} = \left(\frac{\delta F}{F}\right)^{1/2} . \tag{12.24}$$

If the planet is much less massive than the star, its orbital semimajor axis a_B is given by Kepler's third law (equation 12.16), and its speed (assuming a circular orbit) is

$$v_B \approx \frac{2\pi a_B}{P} \approx 30 \text{ km s}^{-1} \left(\frac{M_A}{1 M_\odot}\right)^{1/3} \left(\frac{P}{1 \text{ yr}}\right)^{-1/3} . \tag{12.25}$$

If we are seeing the exoplanet's orbit exactly edge-on, then between the time of first contact (t_1) and the time of second contact (t_2), the planet moves a distance equal to its own diameter. In this case, the radius of the planet is

$$R_B = \frac{1}{2} v_B (t_2 - t_1) \approx 54{,}000 \text{ km} \left(\frac{M_A}{1 M_\odot}\right)^{1/3} \left(\frac{P}{1 \text{ yr}}\right)^{-1/3} \frac{t_2 - t_1}{1 \text{ hr}} . \tag{12.26}$$

Similarly, between the time of first and third contact, if the exoplanet's orbit is edge-on, the planet moves a distance equal to the *star's* diameter, enabling the radius of the star to be calculated as well.

For an arbitrary exoplanet, our probability of seeing it in transit across its star is small. This is because a transit can be seen only if we are very close to the orbital plane of the planet. Figure 12.8 shows a star (A) and planet (B) separated by a distance a. The star is at a distance d from an observer and the planet is at a distance d'; however, when $d \gg a$, as we expect when observing exoplanets, we can make the approximation $d' \approx d$. The angular radius of the star, as seen by the observer, will be $\theta_A \approx R_A/d$. The angular radius of the planet will be $\theta_B \approx R_B/d' \approx R_B/d$. The observer will see a transit when the angle θ between the planet's center and the star's center satisfies the relation

$$\theta \le \theta_A + \theta_B \approx \frac{R_A + R_B}{d} . \tag{12.27}$$

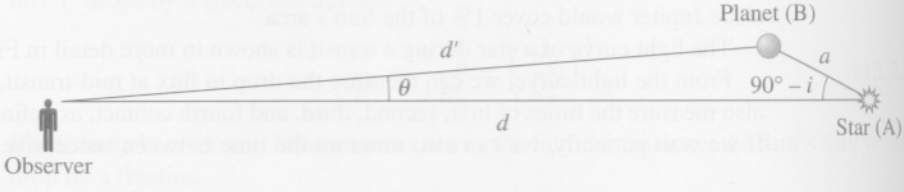

FIGURE 12.8 The geometry that determines whether an observer will see a planet (B) transit a star (A).