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RATIO OF LINE INTENSITIES IN HELIUM-LIKE IONS AS A DENSITY INDICATOR

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ABSTRACT

The ratio R of the forbidden- to intercombination-line intensities in helium-like ions has been evaluated as a function of temperature. The effects of cascades to the $n = 2$ levels following recombination and collisional excitation of higher levels have been considered. Both the temperature variation and the effect of cascades are important for an accurate determination of R. The use of R as a density indicator in solar active regions and flares is discussed, and the theoretical results are compared with observations. We also discuss how in thermal cosmic X-ray sources this method may give information on the electron density in the source or on the ultraviolet flux in the vicinity of the hot plasma.

I. INTRODUCTION

It has been pointed out by Gabriel and Jordan (1969*a*) that in certain cases the ratio R of the intensity of the $2\,^3S \rightarrow 1\,^1S$ to the intensity of the $1\,^3P \rightarrow 1\,^1S$ transitions in helium-like ions can be used to estimate the electron density N of a hot plasma. When applied to the soft X-ray spectrum of the solar corona, this procedure has yielded the result that the highest electron densities occur in the regions having the highest temperature. For example, it has been found that $N \sim 10^9$ cm^{-3} at $T \sim 2 \times 10^6\,°$ K while $N \sim 10^{12}$ cm^{-3} at $T \sim 10^7\,°$ K (cf. Walker and Rugge 1970; Gabriel and Jordan 1969*b;* Freeman *et al.* 1971), just the opposite of what one would expect on the basis of approximate pressure equilibrium in the lower solar corona. If magnetic pressure is to be important in this context, magnetic fields as high as 1000 gauss are required. In view of the great potential importance of this method of density determination, we have re-evaluated the ratio R for ions from C v to Fe xxv, using the most accurate atomic data available.

In § II the basic equations and assumptions concerning the evaluation of R are discussed. Although previous investigations (cf. Gabriel and Jordan 1969*b*) have treated the ratio of collisional population rate of the $2\,^3S$ state from the ground state to that of the $2\,^3P$ states from the ground state as a free parameter to be determined observationally along with N, we explicitly evaluate this ratio as a function of temperature, including cascades. Finally, in § III two possible applications of this method are discussed. First, the applicability to solar active regions and solar flares is considered, and the theoretical results obtained here are compared with observations. We also discuss how in thermal cosmic X-ray sources this method may give information on the electron density in the source or on the ultraviolet flux in the vicinity of the hot plasma.

II. BASIC EQUATIONS AND ASSUMPTIONS

That portion of the energy-level diagram of the helium isoelectronic sequence which is relevant for our considerations is shown in Figure 1. Also shown are the processes

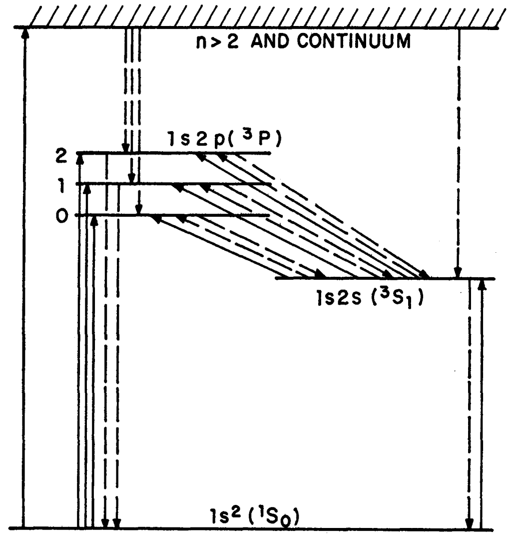

FIG. 1.—Energy-level diagram for the triplet states of helium-like ions. Slanted and vertical lines correspond to ultraviolet and X-ray transitions, respectively. Solid lines denote collisional transitions; broken lines, radiative transition.

which are important for determining the populations of the various levels, with solid lines denoting collisional and broken lines denoting radiative transitions. Note that the transition from the $2\,^3P_0$ state to the ground state is strictly forbidden in the absence of nuclear spin since both states have $J = 0$. It is only through the linear dependence on N of the collisional excitation rate from the $2\,^3S$ to the $2\,^3P$ states that the ratio R of the intensity of the forbidden ($2\,^3S_0 \rightarrow 1\,^1S_0$) line to the intensity of the intercombination lines ($2\,^3P \rightarrow 1\,^1S_0$) depends on the density. The population of the singlet $n = 2$ states has no effect on R and is ignored. For ions heavier than C v the $2\,^1S_0$ lies above the $2\,^3P_1$ state, and has a permitted electric dipole transition to the $2\,^3P_1$ state. This transition rate is always much less than the rate of two-photon decay of the $2\,^1S_0$ state (Drake and Dalgarno 1969; Dalgarno 1971). The short lifetimes of the singlet $n = 2$ states also precludes the possibility that collisional transitions with the triplet states can be an important mechanism for populating the latter.

The steady-state rate equations can then be solved to yield the ratio R (Gabriel and Jordan 1969*b*):

$$R = \frac{I(2\,^3S_1 \rightarrow 1\,^1S_0)}{I(2\,^3P_1 \rightarrow 1\,^1S_0) + I(2\,^3P_2 \rightarrow 1\,^1S_0)}\,, \tag{1a}$$

$$R = \frac{A(2\,^3S_1 \rightarrow 1\,^1S_0)B^{-1}(1 + F - B)}{A(2\,^3S_1 \rightarrow 1\,^1S_0) + (1 + F)[NC(2\,^3S_1 \rightarrow 2\,^3P) + \phi(2\,^3S_1 \rightarrow 2\,^3P)]}\,, \tag{1b}$$

where

$$F = \frac{C(1\,^1S_0 \rightarrow 2\,^3S) + K_c(^3S) + K_r(^3S)}{C(1\,^1S_0 \rightarrow 2\,^3P) + K_c(^3P) + K_r(^3P)} \tag{2}$$

and

$$B = \frac{1}{3}\,\frac{A(2\,^3P_1 \rightarrow 1\,^1S_0)}{A(2\,^3P_1 \rightarrow 1\,^1S_0) + A(2\,^3P_1 \rightarrow 2\,^3S_1)} + \frac{5}{9}\,\frac{A(2\,^3P_2 \rightarrow 1\,^1S_0)}{A(2\,^3P_2 \rightarrow 1\,^1S_0) + A(2\,^3P_2 \rightarrow 2\,^3S_1)}\,. \tag{3}$$

In equations (1)–(3), $A(i \rightarrow j)$ is the spontaneous transition probability from state i to state j, N is the electron density, $C(i \rightarrow j)$ is the collisional excitation rate coefficient from level i to j, $\phi(i \rightarrow j)$ is the photoexcitation rate, and $K_r(i)$ and $K_c(i)$ are terms which take into account the population of level i through recombination and cascades from higher levels following excitation, respectively. In deriving equation (1), it is assumed that the ^3P states are populated according to statistical weight. Equation (3) differs from the corresponding equation given by Gabriel and Jordan (1969*b*) in that the terms $A(2\,^3P_J{}^3 \rightarrow 2\,^3S_1)$ appear in the denominator. They assumed that this quantity is independent of J; the results quoted in Table 1 show that this is not so for large Z where relativistic corrections to the transition frequency become important. Note, however, that R, and indeed the absolute intensities making up R, are all independent of $A(2\,^3P_0 \rightarrow 2\,^3S_1)$.

In order to study the usefulness of equation (1b) for determining the densities in hot plasmas, we rewrite it in the following form:

$$R = R_0 \Big/ \left(1 + \frac{\phi}{\phi_c} + \frac{N}{N_c}\right), \tag{1c}$$

where

$$R_0 = \frac{1+F}{B} - 1\,, \quad \phi_c = \frac{A(2\,^3S_1 \rightarrow 1\,^1S_0)}{1+F}\,, \quad \text{and } N_c = \frac{A(2\,^3S_1 \rightarrow 1\,^1S_0)}{(1+F)C(2\,^3S_1 \rightarrow 2\,^3P)}\,. \tag{4}$$

The quantities R_0, ϕ_c, and N_c depend only on atomic parameters and the electron temperature in the source. From equation (1c) one sees that it may be possible in some cases to estimate the far ultraviolet flux in the line emitting region by estimating $\phi(2\,^3S_1 \rightarrow 2\,^3P)$.

The values for $A(i \rightarrow j)$ as well as the branching ratio B are listed in Table 1. The transition rates $A(2\,^3P_2 \rightarrow 1\,^1S_0)$ are taken from the work of Drake (1969, 1971*a*), while the transition rates $A(2\,^3P_J \rightarrow 2\,^3S_1)$ are taken from Dalgarno and Drake (1972). The $A(2\,^3S_1 \rightarrow 1\,^1S_0)$ values are obtained from Drake (1971*b*); note that this transition rate scales as Z^{10} (Griem 1970) as opposed to the Z^8 dependence derived earlier by Griem (1969). The $2\,^3S_1$ state can also decay to the ground state via a two-photon emission process, but since the rate of two-photon decay is much less than the rate of single-photon decay (Drake, Victor, and Dalgarno 1969; Bely and Faucher 1969), it can be safely ignored. Finally, the $A(2\,^3P_1 \rightarrow 1\,^1S_0)$ transition rates listed in Table 1 are taken from Drake and Dalgarno (1969). For $Z \lesssim 18$ this transition rate scales approximately as Z^{10}, but must eventually flatten to Z^4 in the limit of complete spin-orbit mixing. Since $A(2\,^3P_1 \rightarrow 1\,^1S_0) \gg A(2\,^3P_1 \rightarrow 2\,^3S_1)$, equation (3) shows that for ions heavier than Ne IX, B is almost completely insensitive to the precise value of $A(2\,^3P_1 \rightarrow 1\,^1S_0)$.

To obtain the collisional excitation rates from the $2\,^3S$ to the $2\,^3P$ states, we used

TABLE 1

TRANSITION RATES FOR HELIUM-LIKE IONS*

Ion	$A(^3P_2 \rightarrow {}^1S_0)$ [s^{-1}]	$A(^3P_1 \rightarrow {}^1S_0)$ [s^{-1}]	$A(^3S_1 \rightarrow {}^1S_0)$ [s^{-1}]	$A(^3P_1 \rightarrow {}^3S_1)$ [s^{-1}]	$A(^3P_2 \rightarrow {}^3S_1)$ [s^{-1}]	B
C V	2.62 (4)	2.84 (7)	4.86 (1)	5.70 (7)	5.72 (7)	0.111
N VI	1.03 (5)	1.40 (8)	2.53 (2)	6.86 (7)	6.92 (7)	0.225
O VII	3.34 (5)	5.53 (8)	1.04 (3)	8.05 (7)	8.20 (7)	0.293
Ne IX	2.27 (6)	5.40 (9)	1.09 (4)	1.06 (8)	1.11 (8)	0.338
Mg XI	1.06 (7)	...	7.24 (4)	1.34 (8)	1.47 (8)	0.370
Al XII	2.09 (7)	...	1.66 (5)	1.50 (8)	1.69 (8)	0.391
Si XIII	3.88 (7)	...	3.56 (5)	1.66 (8)	1.95 (8)	0.425
S XV	1.18 (8)	...	1.41 (6)	2.04 (8)	2.62 (8)	0.506
Ca XIX	7.95 (8)	...	1.38 (7)	2.97 (8)	4.97 (8)	0.675
Fe XXV	7.98 (9)	...	2.00 (8)	4.93 (8)	1.49 (9)	0.801

* Number in parentheses denotes exponent of 10.

the calculations of Blaha (1971*a*, *b*), who included the effects of collisions with heavy particles. At the large temperatures necessary to maintain high-Z elements in a helium-like state of ionization, transitions caused by protons and helium nuclei make a significant contribution to this collisional excitation rate. These rates were then evaluated at T_M, that temperature at which the forbidden- and intercombination-line emission from the ion under consideration reaches a maximum (Tucker and Koren 1971). Both T_M and $C(T_M)$ are listed in Table 2. Of the 10 ions considered here, Blaha's (1971*a*, *b*) calculations included all but N VI, Al XII, and S XV. However, since the collisional excitation rate from the 2 ^3S to the 2 ^3P states is a smooth function of T and Z ,we interpolated to obtain $C(T_M)$ for the above three ions, a procedure that probably results in errors no larger than $\sim$20 percent.

The collision rate coefficients for the $C(1\,^1S \to 2\,^3P)$ and $C(1\,^1S \to 2\,^3S)$ transitions were computed by using the collision strengths given by Burgess, Hummer, and Tully (1970) in the limit of nuclear charge $Z \to \infty$, values which are in good agreement with the results of Beigman and Vainshtein (1967) for C V.

To estimate the $K_c(i)$ term in equation (2), the cross-sections for the transitions to the $n\,^3S$ and $n\,^3P$ levels for $n \geq 3$ were assumed to scale in the same way as for neutral helium, i.e., approximately as $1/n^3$ (cf. Ochkur and Bratsev 1965). The cross-sections for excitation to states with $L > 1$ are less than 1 percent of the 2 ^3S and 2 ^3P cross-sections, so excitation to higher-L states need not be considered. The cascade probabilities were then computed by assuming that the relative transition probabilities for the $n\,^3P \to n'\,^3S$ and $n\,^3S \to n'\,^3P$ transitions are the same as those given by Gabriel and Heddle (1960) for neutral helium, since these transitions are all electric dipole. To a first approximation, an excitation of an $n\,^3P$ leads ultimately to the population of the 2 ^3S level and vice versa. A more accurate treatment yields the following approximate expression for $K_c(2\,^3S)$ and $K_c(2\,^3P)$:

$$K_c(2\,^3S) \approx 0.4e^{-\xi/4}C(1\,^1S \to 2\,^3P) \quad \text{and} \quad K_c(2\,^3P) \approx 0.2e^{-\xi/4}C(1\,^1S \to 2\,^3S)\,, \quad (5)$$

where $\xi = I/kT$ (ξ is generally about 2 or 3), and I is the ionization potential.

In evaluating $K_r(i)$ under steady-state conditions, we note that the number of recombinations equals the number of ionizations. Hence, the ratio of excitations of the 2 ^3S and 2 ^3P states to *all* recombinations is just the ratio of these excitations to ionizations which is approximately (Bely 1970) ξ exp ($\xi/4$). Nearly all the recombinations to triplet levels lead eventually to 2 ^3S and 2 ^3P levels. Dielectronic recombination is the most important recombination process under steady-state conditions near T_M, so almost all the recombinations are to levels with $n > 2$, and the relative rate of popula-

TABLE 2

ATOMIC PARAMETERS AND RESULTS[a]

Ion	$\log_{10}T_M$ [° K]	ξ_M	$1+F_M$	$R_0(\xi_M)$	$C(\xi_M)$ [cm^3 s^{-1}]	$N_c(\xi_M)$ [cm^{-3}]	$\phi_c(\xi_M)$ [s^{-1}]	$\phi_\odot/\phi_c(\xi_M)$	$\phi_*/\phi_c(\xi_M)$[b]
C V.......	5.9	5.0	1.36	11.3	5.3 (−8)	6.7 (8)	3.57 (1)	7.5	2.7 (6)
N VI......	6.1	4.5	1.38	5.13	3.1 (−8)	5.9 (9)	1.83 (2)	1.5 (−1)	5.0 (5)
O VII.....	6.3	3.9	1.42	3.85	2.2 (−8)	3.4 (10)	7.32 (2)	3.5 (−3)	1.2 (5)
Ne IX.....	6.5	4.0	1.41	3.17	1.2 (−8)	6.4 (11)	7.73 (3)	...	9.9 (3)
Mg XI....	6.8	3.0	1.49	3.03	7.9 (−9)	6.2 (12)	4.86 (4)	...	1.4 (3)
Al XII....	6.85	3.2	1.47	2.76	6.7 (−9)	1.7 (13)	1.13 (5)	...	5.7 (2)
Si XIII....	6.95	3.0	1.49	2.51	6.0 (−9)	4.0 (13)	2.39 (5)	...	2.6 (2)
S XV......	7.15	2.5	1.54	2.04	4.9 (−9)	1.9 (14)	9.16 (5)	...	6.0 (1)
Ca XIX...	7.4	2.3	1.57	1.33	4.2 (−9)	2.1 (15)	8.79 (6)	...	5.4 (0)
Fe XXV...	7.7	2.0	1.62	1.02	2.7 (−9)	4.7 (16)	1.23 (8)	...	2.9 (−1)

[a] Number in parentheses denotes exponent of 10.

[b] ϕ_* is calculated for a blackbody at $T = 10^5$ ° K with a dilution factor of 0.5.

tion of the 2 ^3S and 2 ^3P levels is determined by the cascade probabilities. These have been computed by Robbins (1968) for helium; from his results we find that the rate of population of the 2 ^3S level is roughly twice that of the 2 ^3P level. Therefore, for the $n = 2$ triplet levels,

$$K_r(L) = \tfrac{1}{3}(2 - L)\xi^{-1}e^{-\xi/4}[C(1\,^1S \rightarrow 2\,^3S) + C(1\,^1S \rightarrow 2\,^3P)]\,. \tag{6}$$

Thus, from equation (2),

$$F(\xi) = \frac{3\xi H(\xi)\exp(\xi/4) + 1.2\xi + 2H(\xi) + 2}{3\xi\exp(\xi/4) + 0.6\xi H(\xi) + H(\xi) + 1}\,, \tag{7}$$

where

$$H(\xi) = \frac{C(1\,^1S \rightarrow 2\,^3S)}{C(1\,^1S \rightarrow 2\,^3P)}\,. \tag{8}$$

Both $H(\xi)$ and $F(\xi)$ are plotted in Figure 2. $H(\xi)$ is a slowly varying function of ξ, ranging from 0.24 at $\xi = 0$ to 0.20 at $\xi = \infty$. However, as can be seen from Figure 2, $F(\xi)$ is a much more rapidly varying function of ξ, and for $\xi \leq 3$, $F(\xi)$ is more than twice as large as $H(\xi)$. Therefore, recombination and cascades make a significant contribution to F. Values of F evaluated at T_M for the various ions are listed in Table 2.

In the case of solar flares, it is possible that the cooling time can be shorter than the recombination time, in which case steady-state conditions do not apply, and the gas can be more ionized for a given temperature T than in a steady state. Then collisional excitation and dielectronic recombination are unimportant, and the value of F is fixed by the *total* recombination coefficient including cascades. The calculations by Robbins (1968) yield a value of $F \approx 1.4$. The difference between this value and the one which would be obtained from equation (7) by neglecting collisional excitation from the ground state is due to the contribution of recombinations directly to the 2 ^3P level. These are unimportant for dielectronic recombination but contribute about 40 percent of the total recombination coefficient for the 2 ^3P state for radiative recombination. The large difference between the value of F for cooling and steady-state plasmas suggests that the ratio R can be used to investigate the departure from steady-state conditions in hot plasmas.

In Table 2 we have listed values for R_0, N_c, and ϕ_c evaluated at $T = T_M$. Note that $R_0(\xi_M)$ is a decreasing function of Z, varying from 11.3 for C v to 1.02 for Fe xxv. The values of N_c for the various ions span the range of densities expected in both the solar corona and thermal cosmic X-ray sources. Assuming that observation of the line ratio

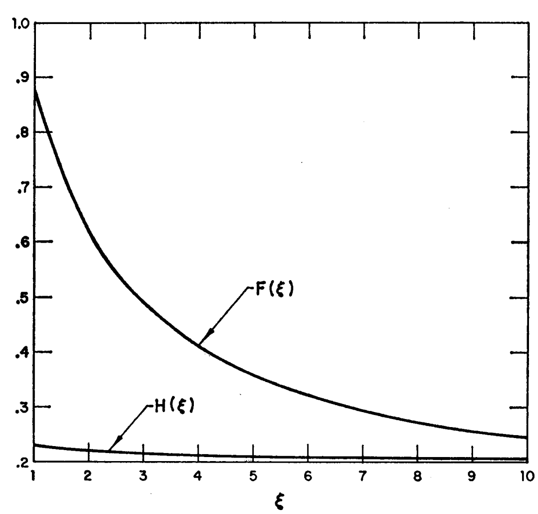

FIG. 2.—$F(\xi)$ and $H(\xi)$ as functions of $\xi = I/kT$

R for a particular ion always occurs from a hot plasma with temperature T_M, it follows that $R > R_0$ implies that non–steady-state conditions such as those in solar flares may prevail, while $R < R_0$ implies that collisional or photoexcitation from the $2\,^3S$ to the $2\,^3P$ levels is becoming important.

Unfortunately, the correct picture is not quite so simple since the temperature of the hot plasma need not equal T_M. To illustrate this point, Figure 3 shows a plot of the emissivity of the appropriate oxygen lines as a function of temperature (Tucker and Koren 1971). Also shown in the figure are the temperature dependences of R_0, N_c, and that value of N which would be obtained from equation (1c) is $\phi = 0$ and $R = 2$. For O VII $T_M \approx 2 \times 10^6\,^\circ$ K, while the emissivity is half its maximum for $T_1 \approx 1.3 \times 10^6\,^\circ$ K and $T_2 \approx 2.9 \times 10^6\,^\circ$ K. Then, if a hot plasma has a temperature $T_1 \leq T \leq T_2$, it follows that $3.5 \leq R_0 \leq 4.2$; and if R is observed to be equal to 2, then $2.6 \times 10^{10} \leq N \leq 3.6 \times 10^{10}$ cm^{-3}. Thus, these results can be somewhat uncertain unless the temperature is known. For a homogeneous source the temperature can be obtained from a detailed spectrum, whereas for inhomogeneous sources the ratio of the lines formed by excitation of the $n = 2$ and $n = 3$ states of the helium-like ion in question is the only reliable indicator.

By applying the principle of detailed balance, the photoexcitation rate ϕ in equation (1c) can be expressed in terms of u_ν, the spectral energy density of radiation in the hot plasma at the appropriate $2\,^3P \rightarrow 2\,^3S$ frequency:

$$\frac{\phi}{\phi_c} = \frac{3(1 + F)c^3}{8\pi h\nu^3} \frac{A(2\,^3P \rightarrow 2\,^3S)}{A(2\,^3S_1 \rightarrow 1\,^1S_0)} u_\nu . \tag{9}$$

This equation assumes the $A(2\,^3P_J \rightarrow 2\,^3S_1)$ rates and the $\nu(2\,^3P_J \rightarrow 2\,^3S_1)$ to be equal, which is not the case for large Z. However, using average values (over J) in equation (9) still yields a good enough approximation for our purposes.

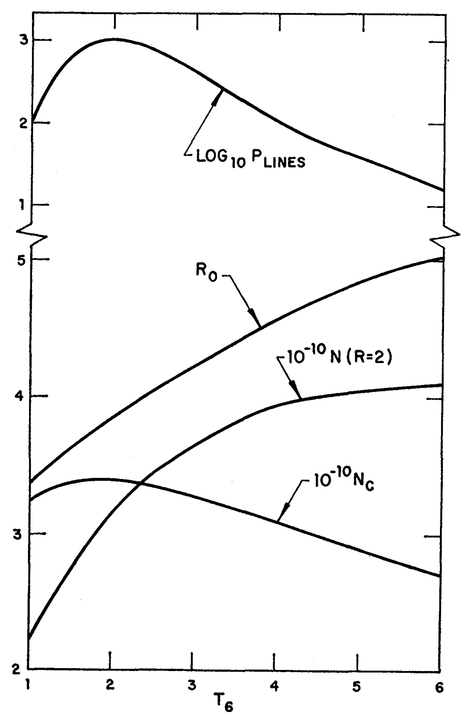

FIG. 3.—Temperature dependence of R_0, N_c, and the N implied by an observed value of $R = 2$ for O VII. Also shown is the temperature dependence of the sum of the intercombination- and forbidden-line intensities (arbitrary units). N and N_c are in units of cm^{-3}, while T_6 is the electron temperature in units of $10^6\,^\circ$ K.

III. APPLICATION TO HOT PLASMAS IN ASTROPHYSICS

a) The Sun

A fair number of observational results are available on the ratio R in solar active regions and flares (Fritz *et al.* 1967; Evans and Pounds 1968; Freeman and Jones 1970; Walker and Rugge 1970; Rugge and Walker 1970, 1971; Doschek *et al.* 1971; Batstone *et al.* 1970). The observed values show considerable scatter and are for the most part below the low density limit R_0, even when one takes into account the variation of R_0 with temperature. In the case of C v, $R < R_0$ can be readily understood in terms of photoexcitation, as seen from Table 2. At $T = T_M$, and with blackbody radiation with $T_\odot = 5000°$ K and a dilution factor of 0.5, $\phi/\phi_c = 7.5$ for the Sun, so $R = R_0/8.5 = 1.3$, to be compared with the observed values of 1 and 2. The observations for the other ions have been interpreted as a density effect leading to density estimates as high as 3×10^{14} cm^{-3} for the S xv ion (Freeman *et al.* 1971). However, Rugge and Walker (1971) point out that there may be a systematic error in their estimates of the intensity of the flux in the intercombination line of oxygen due to the extended wings of the resonance line caused by the resolution of their instrument. They suggest that the intercombination line intensity should be reduced by about 15 percent to correct for this effect. Then their data yield for oxygen $R \approx 3.92$ for a number of measurements made in 1966/1967 and $R \approx 3.78$ for 1969 data, values which are in good agreement with the low-density limit given in Table 2. The data on other ions are probably subject to the same uncertainty. Furthermore, satellite lines from autoionizing states may further confuse the picture, so we feel that it is premature at this time to conclude that density effects in R are in fact being observed in the solar corona.

b) Cosmic X-Ray Sources

With the use of Bragg crystal spectroscopy, it is to be hoped that X-ray lines will soon be resolved in cosmic X-ray sources. Then, since the values of N_c given in Table 2 vary up to $N_c = 4.7 \times 10^{16}$ cm^{-3} for iron, one might hope to use the line ratio R to determine the electron density in thermal X-ray sources. However, the ratio R can really only be used to determine the quantity $(\phi/\phi_c) + (N/N_c)$, so that an observation of R actually gives a linear relationship between the electron density and the spectral energy density of radiation in the source.

It is easy to estimate the photoexcitation rate due to radiation from the hot plasma itself. The radiation spectrum at the ultraviolet frequencies considered here should be relatively flat as in bremsstrahlung emission, since most of the bound-bound and free-bound emission occurs near $h\nu \sim kT$. Then, assuming an isothermal spherical source with optical depth less than unity and assuming the total emission is given by the bremsstrahlung formula (which is a good approximation for $T \gtrsim 10^{7}$ ° K and an underestimate for smaller temperatures), the spectral energy density of radiation becomes

$$u_\nu \approx \eta \left(\frac{N}{N_c}\right)^{4/3} \left(\frac{T_M}{T}\right)^{2/3} \left(\frac{P}{10^{36}}\right)^{1/3} \text{ ergs Hz}^{-1} \text{ cm}^{-3}\,, \tag{10}$$

where

$$\eta = 5.1 \times 10^{-28} N_c^{4/3} T_M^{-2/3} \tag{11}$$

and P is the total luminosity of the source in ergs s^{-1}. For $T = T_M$ and $N = N_c$, equation (9) then implies that ϕ/ϕ_c varies from about 0.01 for C v to 0.4 for Ca XIX and 1 for Fe XXV. However, for large Z, these parameters do not give an optical depth less than unity; indeed, the optical depth due to Thomson scattering is

$$\tau \approx 3.3 \times 10^{-4} N^{1/3} T^{-1/6} (P/10^{36})^{1/3}\,, \tag{12}$$

which is greater than 1 for $Z > 15$. This would tend to increase ϕ/ϕ_c until the lines become so broadened that they are unresolvable. Nevertheless, since equation (10) implies that $\phi/\phi_c \propto (N/N_c)^{4/3}$, one can conclude that so long as N does not exceed N_c

too greatly, the effect of photoexcitation due to thermal radiation from the plasma itself is not as important as the effect of collisional excitation in decreasing the value of R for all ions except Fe XXV and perhaps Ca XIX.

However, photoexcitation can become significantly more important than collisional excitation if a thermal X-ray source surrounds a mildly hot underlying body such as a white dwarf. Then, the ultraviolet radiation spectrum can be approximately thermal in which case

$$u_\nu = \frac{4\pi h\nu^3 R^2}{c^3 R_*^2}(e^{h\nu/kT} - 1)^{-1}, \tag{13}$$

where T_* and R_* are the temperature and radius of the underlying body, while R is the distance of the hot plasma from the center of the star. For $R = R_*$ and $T_* = 10^5\,°$ K, ϕ/ϕ_c has been evaluated and is listed in Table 2. The ratio ϕ/ϕ_c is very large for small Z and approaches unity for the large-Z elements. Therefore, the existence of an underlying object with an ultraviolet brightness temperature of $10^4\,°$–$10^5\,°$ K could be detected by observing R.

In conclusion, since photoexcitation due to radiation from the plasma itself cannot be important except for large Z (iron) or unless $R \ll R_0$, the observation of the line ratio R in a thermal X-ray source less than, but not orders of magnitude less than, R_0 implies either that $N \gtrsim N_c$ or that there exists an additional source of ultraviolet radiation near or in the source.

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