A Optical Depth in a Porous Medium

Let us derive here some simple relations for how the absorption in a wind outflow could be affected by the "porosity" of the medium. We begin by writing the expression for radial optical depth in a smooth stellar wind,

$$\tau(r) = \int_r^\infty dr' \, \kappa\rho(r') = \frac{\kappa\dot{M}}{4\pi v_\infty R_*} \frac{R_*}{r} \equiv \tau_* \frac{R_*}{r} \equiv \frac{R_1}{r} \, , \tag{1}$$

where for simplicity we've assumed a constant flow speed v_∞, and have defined the characteristic wind optical depth τ_* and the unit optical depth radius $R_1 = \tau_* R_*$ in terms of the mass loss rate $\dot{M}$ and stellar radius R_*.

Following the general approach of Owocki, Gayley, and Shaviv (2004), consider then a porous wind in which the microscopic opacity is modified into an "effective opacity",

$$\kappa_{eff} \approx \frac{\kappa}{1+\tau_b} \, , \tag{2}$$

where $\tau_b = \kappa\rho H$ is the optical depth of individual clumps or blobs, with $H = \ell/f$ the "porosity length" for clumps of scale ℓ and volume filling factor f. Note that optically thin blobs ($\tau_b \ll 1$) recover the non-porous limit, $\kappa_{eff} \rightarrow \kappa$, whereas the limit of optically thick blobs ($\tau_b \gg 1$) has an effective opacity given by the ratio of their area ℓ^2 to mass $m_b (= \ell^3\rho/f = \ell^2\rho H)$, i.e. $\kappa_{eff} \rightarrow 1/\rho H$. The effective opacity (2) thus retains the proper asymptotic limits, with however a somewhat simpler general form than posited by Owocki et al. (2004) (cf. their eqn. 35).

Indeed a key advantage of this simpler form is that it can be integrated analytically to obtain the effective optical depth

$$\begin{aligned} \tau_{eff}(r) &= \int_r^\infty dr' \kappa\rho(r') / \left(1 + \kappa\rho(r')H\right) & (3) \\ &= R_1 \int_r^\infty \frac{dr'}{r'^2 + HR_1} & (4) \\ &= \sqrt{H/R_1} \, \arctan\left(\sqrt{HR_1}/r\right) . & (5) \end{aligned}$$

The requirement $\tau_{eff}(R_{b1}) \equiv 1$ then defines the radius R_{b1} at which the effective optical depth of this *porous* wind becomes unity. After solving this, the associated porosity reduction in the radius of unit optical depth can be written

$$\frac{R_{b1}}{R_1} = \frac{h}{\tan h} \, . \tag{6}$$

where $h \equiv \sqrt{H/R_1}$. This ratio declines from unity at small h to small values as $h \to \pi$, crossing a value of 1/2 for $h = 1.17$.

A central result here is that a significant reduction in the unit optical depth radius (which is what is required to make the wind more transparent) requires $h \approx 1$, implying a quite large porosity length, comparable to the unit optical depth radius in the smooth wind, i.e. $H \approx R_1$. Since $H \equiv \ell/f$, this requires either very large blobs, $\ell \lesssim R_1$, or very small filling factors, $f < \ell/R_1$, or some combination of these.

In this regard, we note that structure arising from the line-driven instability has a typical spatial scale on the order of the mean-wind Sobolev length, $L \equiv v_{th}/(dv/dr) \approx (v_{th}/v_\infty)R_* \approx R_*/100$, and a typical volume filling factor of order $f \gtrsim 1/10$. This implies an associated porosity length $H \lesssim R_*/10$, about a factor 100 too small to reduce the absorption for a typical optically thick case with $\tau_* \approx 10$, and thus $R_1 \approx 10R_*$.

It thus seems rather unlikely that the structure arising from the line-driven instability could lead to a substantial porosity that allows the wind to be more transparent to absorption of, e.g. X-rays. If porosity is a contributing factor in the apparent reduced absorption of such X-rays, it requires structure on a relatively large spatial scale, or a very small volume filling factor.

Another key general point here is that the "porosity" effect in reducing absorption is quite distinct from the "clumping" effect that can enhance processes that scale with density-squared. The latter depends only on the volume filling factor f, while the former depends on this filling factor *and* the characteristic spatial scale of the structure, in the combination characterized by the porosity length $H = \ell/f$. This distinction stems from the fact that the porosity effect requires the individual blobs or clumps to be optically thick, so that material in the front side of the blob can effectively "hide" or "shadow" other material within the blob, thus reducing the overall effective opacity of the medium.

Such a requirement makes porosity more effective in "inside-out" radiative transport, such as in reducing the effective coupling between radiative and matter in a stellar envelope and atmosphere. As shown in Owocki et al. (2004), this can, for example, allow a super-Eddington star to have a quasi-steady wind from a bound surface. In contrast, it seems inherently more difficult for porosity to play a role in "outside-in" problems, such as the transparency of wind X-ray emission as viewed by an external observer, which is the case of principal concern in this paper.