Astronomy 128: Galaxies and Galactic Structure

Week 4, Thursday, February 10

**Topic:** Galactic dynamics: motion of stars under the influence of gravity

This week, we continue to look at motions of stars within the Galaxy, but from the other direction: we’ll be looking at various gravitational potentials, density distributions, and rotation curves, and considering more specifically how they are inter-related. This, of course, involves gravity and the first part of the reading and problems will be a refresher (hopefully) on gravitational potentials, conservation of energy, and conservation of angular momentum. Then we’ll see that a given gravitational potential predicts the rotational speed of circular orbits as a function of radius. This is of immense value, as it ties an observable (rotation curve) to the mass profile of the galaxy. We also spend some time on determining what is more important in determining a star’s trajectory: the smooth overall galactic potential or the sharp potential of the nearest stars. The answer depends on the scale of the system, but for galaxies, the smooth potential is certainly much more important. For clusters, the story is somewhat different, and we’ll see that star-star interactions will cause some clusters to evolve significantly over the age of the Galaxy.

**Break:** Victoria

**Reading:**

Read the first part of Chapter 3 of Sparke & Gallagher (up through section 3.2). You will also want to review some vector calculus including the gradient ($\nabla$) and Laplacian ($\nabla^2$) in spherical coordinate; your Physics 8 notes or book might be a good place to look. We’ll also be using some results that you saw in Physics 7 and 8, including Newton’s results that a spherical mass distribution outside your orbit exerts no force, and a spherical mass distribution inside your orbits acts like a point mass. (You saw these results for the electric field in Physics 8, too—the result is the same, since both forces are $1/r^2$.)

**Problems:**

1. Come to class with at least one written question on the reading. (Please also write down any of these questions on the written assignment you turn in, so that I can get a
chance to see them before class. If you come up with extra questions after that, that’s fine, but if you’ve come up with them already by Wednesday, please write them down.

2. *(Left over from last week.)* Take a look at the major paper on the MACHO project for looking at microlensing by halo objects, Alcock et al. ApJ 542, 281 (2000). Everyone should at least read the abstract and look through the figures; in addition, Vernon will give a roughly 10-minute summary presentation on the paper, focusing on what we can now say about how much of the dark matter halo may be in the form of MACHOs.

3. SG 3.2.

4. SG 3.3.

5. SG 3.5. It’s not quite clear why SG define free-fall time the way they do here, as the *approximate* time for a constant-density cloud to collapse. Show that the actual free-fall collapse time of a constant-density cloud is \( t_f = \sqrt{\frac{3\pi}{32G\rho}} \). Hint: you can do this using Gauss’ law, or using Kepler’s third law and considering a radial in-falling trajectory as a *very* elliptical orbit.

6. SG 3.6.

7. SG 3.11. Be sure to look at the erratum for this problem to correct the result you’re supposed to find!

8. SG 3.13. Once you work out the formula to use, you may find it easier to type the satellite data into a text file and write a short program to do the actual calculations and averaging.


10. *(Binney & Merrifield, Problem 6.8)* See Figure 6.29 in Binney & Merrifield. Assuming that the drop shown there for the number of open clusters with age arises from clusters dispersing into the field, and that each cluster initially contained about 1,000 stars, estimate the total number of stars per square parsec (i.e., the surface density of stars) that these clusters have contributed to the field. If these stars end up distributed in a layer 100 pc thick (comparable to the typical distances of open clusters from the Galactic plane), estimate the number density of these stars in the solar neighborhood. What fraction of nearby stars were born in such clusters? Why might this calculation significantly under-represent the total contribution made by clusters and associations to the field?