Astro 121, Spring 2016
Week 14 (April 27)

Topic: Gravitational wave detection
Break: Katie

An overall note on this assignment: I've never taught this topic before, so this week's seminar reading and problems will be structured somewhat differently structured than previous weeks. I've outlined some problems below to do some representative calculations and to explain some key concepts, but part of what will be doing is just working through together trying to understand this overall problem. So I'll be counting on you to note down questions that come up as you're going through the reading, and also to do a little bit more digging into other sources along the way to try to understand anything that isn't clear. If you do find other sources that are helpful, please make a note of what they are since my hope is to assemble a better set of resources for learning about this topic – it's clear that it will continue to be important in astronomy going into the future.

Reading for next week.

- Kitchin, *Astrophysical Techniques*, has a short section (Sec. 1.6 in the 5th edition) on gravitational wave detection.

- Carroll, *Spacetime and Geometry: An Introduction to General Relativity*, Section 7.7 (pp. 315–320). The level of this textbook is mostly beyond the scope of our course, but this section is a nice discussion of gravitational wave detection, in Sean Carroll's characteristically clear writing.

- "LIGO and the Detection of Gravitational Waves," by Barry C. Barish and Rainier Weiss, *Physics Today* October 1999, pp. 44–50.

- "Detection of gravity waves," by Gary W. Spetz, *The Physics Teacher*, May 1984, pp. 282–287. This pre-dates the building of LIGO, and so it focuses equally on interferometric and resonant bar detectors. We can skip over most of it, but one interesting part is where it explains a bit more about noise sources than the other readings do. Read only p. 284 and the discussion of noise from photon counting and radiation pressure in equations 2 and 3.

- "Observation of Gravitational Waves from a Binary Black Hole Merger," B. P. Abbott *et al*. (LIGO Scientific Collaboration and Virgo Collaboration) *Phys. Rev. Lett*. 116, 061102 – Published 11 February 2016. This is the discovery paper for the first LIGO detection of a gravitational wave event. See below for instructions on commenting / annotating it online.

Important concepts and problems:

0. Carrying over from last week's assignment, I'd like you to get your code working for making informative plots of the Rayleigh-Jeans approximation to the Planck function. Both axes should be clearly labeled, and if necessary, you should use a log scale to be able to see the full range of values displayed. If you run into any coding problems, let me know and I'd be happy to help. But this will only work if you don't do it at the last minute. I think most of you were close, so try to get this done sooner rather than later.

1. *LIGO as an interferometer*. The main detection mechanism in LIGO is called the Michelson interferometer.

    a. Explain the basic detection mechanism. That is, what is changing within the interferometer as a gravitational wave passes, and how is that registered as a signal?

    b. Imagine two waves interfering with each other, with equal amplitudes, but one with a phase shift ϕ with respect to the other: one is $\cos(\omega t)$ and the other is $\cos(\omega t + \phi)$. When we add the two signals together, obviously we will get the greatest combined signal when the condition for constructive interference is met. But what we are looking for is not a steady-state signal, but a *change* in the signal. For what value of ϕ does a *change* in ϕ induce the greatest change in the amplitude of the combined signal? Derive this analytically, but also make a plot to confirm your answer. (Hint: without loss of generality, you can consider only what happens at time $t = 0$.)

    c. Our reading states that a given photon goes back and forth many times within an arm of the interferometer before reaching the detector. Explain how this helps LIGO be more sensitive to gravitational waves.

2. *Sources of noise in LIGO*. In the article by Spetz, he notes that the photon-counting noise is inversely proportional to the square root of the laser power, while the radiation pressure noise is *directly* proportional to the same quantity. Explain why each of these noise sources has the dependence it does.

3. *Signal strength from various events*. In general, a detailed understanding of general relativity is necessary to calculate the strength of gravitational wave events. However, our various readings give some simplifying cases, particularly the case of two compact objects in Keplerian orbit around each other.

    a. Use Kepler's third law to derive Carroll's Eq. 7.194 for the orbital frequency in terms of the Schwarzschild radius. You should get a slightly different constant in the denominator than the factor of 10 he gives there.

    b. Combine this with the strain equation given on page 45 of the Barish and Weiss article to derive Carroll's Eq. 7.195, for the strain in terms of the Schwarzschild radius, source separation, and distance from the observer. (Be careful that the two sources use different notation – stick with Carroll's notation in your derivation.)

    c. For the GW150914 event detected by LIGO, use this equation to calculate the expected strain of the signal received at Earth. Assume the distance and masses given in the LIGO discovery paper, and calculate the strain just before the sources start to merge, i.e. when their Schwarzschild radii are about to overlap.

4. *Periodic vs. one-time signals*. The Barish and Weiss article says, "Periodic sources will have to satisfy a very special set of criteria. The observed signal must exhibit amplitude modulation and Doppler frequency modulation consistent with the effects of the Earth's rotation and revolution around the Sun." Why (and in what way) does the Earth's motion affect the observed gravitational wave signal?

5. *LIGO as a telescope*. LIGO can't be pointed in a particular direction; it's a facility that is fixed to the Earth, in two different locations. However, the Earth is completely transparent to gravitational waves.

    a. Is there any arrival direction from which a gravitational wave would *not* induce a signal in one of the LIGO interferometers? For this part, consider just one site and take the two arms to define the x and y directions, and the perpendicular to the arms to define the z direction.

b. Despite not being movable, explain how LIGO can nevertheless determine information about the direction to the source of a gravitational wave event. What measured quantity is key to determining this directional information? Would this work for steady sources of gravitational waves?

c. When LIGO is able to narrow down the direction to a source, what is the shape of the confidence region on the sky? That is, for a positional uncertainty of, say, 1°, is it a circle on the sky with a 1° radius? Or does it have some other shape? If so, what is that shape, and why?

d. Derive a simple relationship for the positional uncertainty of a source detected by LIGO as a function of the distance between the two LIGO sites and the timing uncertainty with which a signal can be measured.

e. Evaluate your expression (that is, find an angle in degrees) assuming that LIGO has a timing uncertainty of 0.5 ms.

f. When Virgo (a gravitational wave interferometer in Italy) comes on line soon and reaches a comparable sensitivity to LIGO over the next few years, how will this affect the ability of these observatories to localize the position of a source of gravitational waves?

6. Read the discovery paper for GW150914 (citation above). We'll use the online "Nota Bene" platform to annotate it with questions and to note points of interest, and then we'll discuss it and go over questions during seminar.