Topics: Nculeosynthesis and stellar evolution, pt. 1

Reading:

- Read LeBlanc, Ch. 6, the first half (from p. 205 through p. 245).
- Revisit/reread the little bit in LeBlanc Ch. 5 about degeneracy pressure (pp. 189-190) and supplement with Ostlie and Carroll's derivation of the pressure integral (pp. 289-91 in that textbook: pdf on website). Ostlie and Carroll's section on degeneracy pressure is good and worth reading too (it's on pp. 563-69). Degeneracy is important when it comes to post-main-sequence evolution (e.g. the cores of red giants are degenerate) so I've included it in the reading for this week, but we'll spend more time discussing it next week when we talk about white dwarfs and neutron stars.
- Also review LeBlanc on *why* mean molecular weights add like $1/\mu = ...$ (eqs. 5.117, 5.122, 5.123), on the convective criterion (pp. 169-72), and on the physics of period-luminosity relation (pp. 200-02).

Summary of work to be produced:

- There are two warmup problems, due on Thursday at 1 pm: QW1 and QW5.
- Bring solutions to seminar on Friday for all the (non-warm-up) numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope:

We'll go a little deeper and more clearly into a couple of issues from last week that are important for stellar evolution: convection and degeneracy.

There's a lot of LeBlanc reading this week, but it should be less dense than what we've read the last two weeks. I strongly recommend you read the assigned LeBlanc pages straight through once early the week, without taking notes or thinking about the problems you have to solve. After you finish reading, think about the key points and see if you can write them down. Then go back and read more carefully, perhaps with the problems in mind. Though note – there's a lot of good material in the reading that doesn't have corresponding problems. Let's find ways to talk about and maybe even calculate a thing or two related to this material.

Nuclear reactions' physics and relevance are summed up in Fig. 6.5 and 6.1 – and in the magnitude of the nuclear binding force. The outlines of relevant nuclear physics are given in sections 2 and 3. Please try to pull out a few important (and interesting) points and we'll see if we can enumerate them in seminar on Friday.

Particular nuclear reactions dominate based on the temperature, density, and composition in a star's interior. Whichever reaction can most easily (i.e. at the lowest temperature) produce the luminosity required to maintain the temperature gradient demanded by HSEQ will dominate the energy production...until the "fuel" for that reaction is used up.

Elemental abundance variations (that reach the surface, where they can be measured) and neutrino measurements provide observational confirmation of our understanding of the nuclear reactions that power stars, as does the success astronomers have modeling the observable properties of stars.

Questions etc.:

Recall the virial temperature concept and the ability of gravitational contraction to power stars for a long (but not long enough) period of time. The energy of gravity (eq. 6.1) per particle gives a characteristic temperature of a star; and the Sun's virial temperature is indeed a good estimate of the typical interior temperature of the Sun.

Q0 Nuclear reactions and the Coulomb barrier (Fig. 6.5):

Here we'll see how important quantum tunneling is for understanding nuclear fusion reactions at the center of the Sun.

(a) What temperature is required to have two protons fuse without quantum tunneling? To estimate this, assume that the nuclear force can hold them together once they get within 1 femtometer (10^{-15} m) of each other and compute the electrostatic potential energy between two protons at that separation. (Recall that in cgs, Coulomb's constant is unity; compared to $k = \frac{1}{4\pi\epsilon_o}$ in mks units.) Equate that to the energy of a proton with ten times the *average* speed of a Maxwell-Boltzmann distribution (note that that will correspond to 100 times the average particle's energy). The factor of ten (or 100) is somewhat arbitrary, but it allows for the fact that we don't need all the protons to be able to overcome the Coulomb barrier, just a small fraction of them. You may need to look up the expression for the M-B distribution (needed for part (c)) and its average speed as a function of temperature. So, this calculation tells you what *temperature* is required for a small fraction of protons to overcome the Coulomb barrier without any quantum tunneling.

(b) Compare the temperature you just computed to the temperature at the center of the Sun.

(c) Using the full Maxwell-Boltzmann expression, compute the number of particles with a velocity ten times the average to the number with the average velocity. Assume that this approximates the fraction of particles with *at least* ten times the average velocity to *all* particles in the distribution. So, this tells you what *fraction* of the particles in the Sun's core have enough energy to overcome the Coulomb barrier. We assumed above that this small fraction of particles would be enough to account for the fusion reactions in the Sun.

(d) But is it enough? Using Tab. 5.1 and (incorrectly) assuming all the material in the core is hydrogen, compute the absolute number of protons in the Sun's core that are able to overcome the Coulomb barrier and undergo fusion (assuming quantum tunneling is not important).

(e) Comment on the need for quantum tunneling to explain the occurrence of hydrogen-to-helium fusion in the Sun.

Note that we start question numbering with zero here to try to keep book and problem set question numbering aligned.

QW1 Problem 6.1

Q2 Problem 6.2

Q3 Problem 6.3

Q4 Problem 6.4

Why are more massive stars the ones that fuse hydrogen via the CNO cycle on the main sequence (while less massive stars use the p-p chain)?

A first generation of stars in the universe (Population III stars) presumably were composed of only H and He. How would such stars differ in structure and appearance compared to today's more metal-rich stars?

How would a 100 solar mass star forced to use the p-p chain differ from a 100 solar mass star with enough carbon, oxygen, and nitrogen to employ the CNO cycle?

QW5 Problem 6.5

Why are fusion reactions involving heavier elements than hydrogen more difficult to initiate than hydrogen fusion reactions?

Q6 Problem 6.6

What role does degeneracy play in dictating the nature of red giant to horizontal branch (helium-burning) evolution of a solar-type star?

Why does "shell-burning" produce high luminosities? And why does a red giant's position on the HR diagram change gradually with time (stars are said to "climb the red giant branch").

Q7 Problem 6.7

Q8 (this question is an elaboration of what I did at the board during break in week 10):

Note that LeBlanc just states the expression for the degenerate equation of state, and instead spends more time on the density of states in phase space and the maximum number of particles that can occupy a given portion of phase space. Once that density (eq. 5.136) is exceeded, degeneracy pressure is significant.

After reading about the pressure integral, I hope you're convinced that generically (i.e. not just for a perfect gas), the pressure can be approximated by $P = npv/3$, where n is the number density of particles, p is their momentum, and v is their velocity. A full expression involves an integral over the distribution of particle momenta (eq. 10.8 in Ostlie and Carroll) – and indeed, that's how you can derive the perfect gas law from an assumed Maxwellian particle velocity distribution). But for this problem, let's just approximate the pressure as $P = npv/3$ and assume that all particles have the same momentum (and mass, and therefore the same velocity).

Please do a dimensional analysis to make sure that npv has units of pressure.

OK – using this approximation, you are going to derive the equation of state for a non-relativistic degenerate gas (eqns. 5.137, 138). Do it as follows:

(a) Assume that the electrons are in a regular cubic lattice separated from their neighbors by a distance, x. What is the relationship between the number density, n, and x?

(b) Use the Heisenberg uncertainty principle to relate the momentum of the particles to their spatial confinement, x and thus to n.

(c) Use the classical expression that relates velocity and momentum to relate the velocity in the pressure equation to momentum and thus to the particle number density.

(d) Put it all together to derive eq. 5.137 including finding an expression for the constant of proportionality.

(e) Convert it to an expression in terms of mass density to derive eq. 5.138, again, including the constant of proportionality.

(f) Why is electron degeneracy pressure greater than neutron degeneracy pressure for a given mass density?