Topics: Stellar winds part 1

Reading:

- Read Lamers and Cassinelli's *An Introduction to Stellar Winds*, pp. 1 – 30 (end of §2.4).
- Also read the beginning of Ch. 3 of Lamers and Cassinelli, pp. 60 – 65 (very top; end of §3.1.1).

Note that this book is available online, via the library's website. If you're off campus, you'll need to use EZ-Proxy or the College's VPN to access it, but you should, among other things, be able to down load the relevant sections as a pdf. Let me know if you have trouble with this.

Summary of work to be produced:

- There are no warmup problems as such, but you must give or email me your plots for Q2 by Thursday at noon.
- Bring solutions to seminar on Friday for all the numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope:

All stars lose mass from their surfaces, to one extent or another. This more-or-less steady mass loss, or *stellar wind*, has several important aspects:

- It can affect stellar evolution (because for some stars, this mass loss can amount to a significant fraction of their original mass).
- It can deposit significant quantities of mass, momentum, and energy into the interstellar medium (ISM), affecting the structure and energetics and composition of the ISM (and winds are one of the main sites of dust formation in the Universe).
- In binary systems, mass transfer can affect orbital dynamics, evolution, and lead to emission caused by the accretion of wind material onto a companion (nova outbursts, X-ray binaries).
- The Solar wind affects the near-Earth and Solar System environment; and may even be instrumental to the evolution of life on Earth (and for other stars, on the habitability of their exoplanets).
- If stellar winds are strong enough, they can have a significant effect on the appearance of a star's spectrum.
- The physics of stellar winds and the associated observational diagnostics are quite interesting (e.g. radiation-driving) and in some cases subtle, and are directly connected to other interesting phenomena (accretion, jets, active galaxies).
- Many stars undergo impulsive mass loss (not technically a stellar wind, but often involving related physics) that create nebulae (planetary nebula or for massive stars, wind-blown nebulae like η Carina) and have very strong effects on the evolution of these stars and their surrounding ISM.

This week, we will look at the phenomenology of stellar winds, their basic physical properties, including kinematics, and the diagnostics used to study them. We'll begin to look at some aspects of the wind driving physics and theory, and focus on the wind-driving (especially radiation-driving) physics next week.

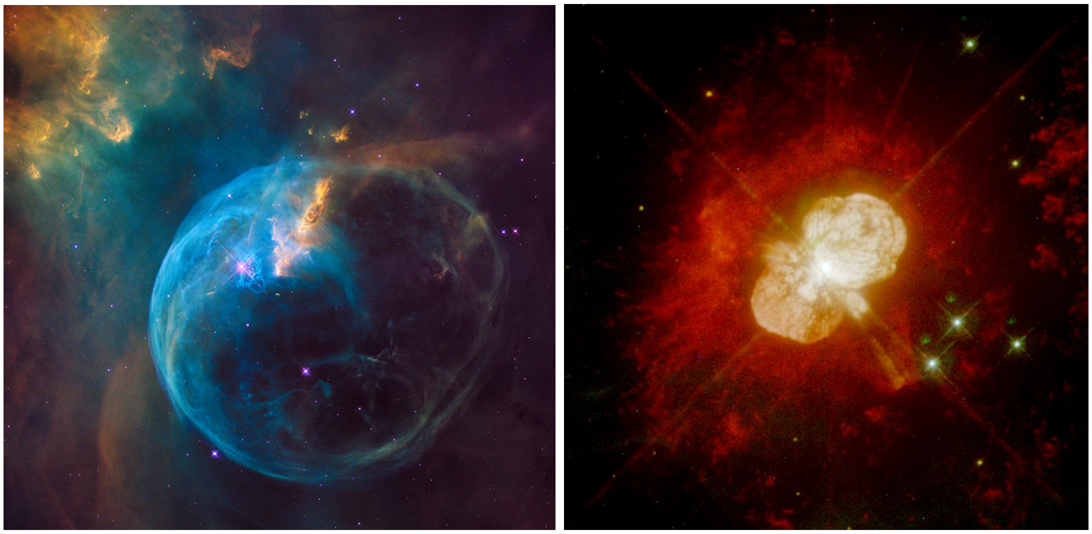

Fig. 1 Stellar mass loss: steady (NGC 7635, left https://apod.nasa.gov/apod/ap160422.html) and impulsive (η Carinae, right https://www.spacetelescope.org/images/opo9409a/).

Questions etc.:

Please note: This week, some questions are long and comprise more than one paragraph. For this week, *all* the questions require written answers. In other words, all the text below is part of a numbered question. *And qualitative questions are questions that require written answers, too.*

Q1 On page 3, the authors mention that there was initially some confusion about the nature of outflows from red supergiants, in which the velocities are only about 5 km s^{-1}. Is there a problem with the terminal velocity of a wind being less than the surface escape velocity of a star? Derive an expression for the escape velocity as a function of radius by equating gravitational potential energy and kinetic energy.

How far (in stellar radii) from a M2 supergiant would material have to get before the local escape velocity is only 5 km s^{-1}? Use the Appendix of Ostlie and Carroll (pdf on website) to find the vital statistics for such a star. For this problem, start by calculating the escape velocity at the star's surface, then write the general expression for $v_{\rm esc}(r)$ in terms of the value at the surface ($v_{\rm esc}(R_{\rm star})$) and some function of $r/R_{\rm star}$. At what $r/R_{\rm star}$ does $v_{\rm esc} = 5$ km s^{-1}?

What does this result imply about where the force (whatever it may be caused by) that pushes the stellar wind is being applied?

How did people in fact figure out that the approximately 5 km s^{-1} outflow in the M star α Her really does exist at large radii?

Q2 How are the wind and photosphere connected? Is it a relatively sudden transition from one to the other? As we've seen, we can define the surface of a star as the radius at which the optical depth is $\tau = 2/3$. As one looks deeper and deeper into a star, the density (and optical depth) goes up and up. Assuming an isothermal atmosphere, use the equation of HSEQ to write down an expression for the density of an atmosphere as a function of radius (probably you'll want to express things in terms of $r - R_{\rm star}$, the height above the "surface"). You can take the acceleration of gravity, g, to be constant, for now.

Using the values for an O5 supergiant (again, from Ostlie and Carroll's appendices), assuming $T = T_{\rm eff}$ and

g constant at and immediately above the photosphere, and assuming a mass density of $\rho = 10^{-5}$ kg m^{-3} = 10^{-8} g cm^{-3} at $r = R_{\rm star}$ and taking $\mu = 0.6$, what is the density scale height of the atmosphere (in km and stellar radii)?

Compare this star's density scale height and g to the Sun's.

Note that the density scale height and pressure scale height have the same value when an atmosphere is isothermal.

Next, derive the density of a *wind* as a function of radius, in terms of the mass-loss rate and using the velocity law given by equation 2.3 of Lamers and Cassinelli, with $r_{\rm o}$ given by the condition that $v_{\rm o} = 10$ km s^{-1}, for the same O5 supergiant. Note that 10 km s^{-1} is roughly the sound speed in the wind.

Assume the mass-loss rate is $\frac{dM}{dt} \equiv \dot{M} = 5 \times 10^{-6}$ M$_\odot$ yr^{-1}, the terminal wind velocity is $v_\infty = 3000$ km s^{-1}, and the wind "velocity law" parameter, $\beta = 1$.

Compare $\rho(r = R_{\rm star})$ for the photosphere and wind and comment on how fast each falls off with distance above the photosphere (looking at the graphs you make, below, could be helpful for answering this last part).

Plot these two functions from $r = R_{\rm star}$ to $r = 5R_{\rm star}$ in log space and also in linear space (use your judgment about axis ranges; perhaps it's not a good idea to require that you can see the atmosphere density value on the right side of your plots). Make a second, expanded plot on the interval $r = 1.00$ to 1.01 R$_{\rm star}$. At what radius are the wind and photospheric densities equal? (You can answer this question by inspecting the plot.) It would be very helpful to make your plots with $r/R_{\rm star}$ as the independent variable on the x-axis.

How many atmosphere scale heights above R$_{\rm star}$ is the point 1.01 R$_{\rm star}$? By what factor does the atmosphere density decrease between these two radii?

Q3 Building on the previous question about the density profile in a hydrostatic atmosphere as compared to a wind, we will apply HSEQ to an *extended* atmosphere, in which gravity is *not* constant. A version of this calculation is what inspired Eugene Parker to posit the existence of the Solar wind in the mid-20th Century, before it had ever been observationally detected. What we are essentially doing here is examining whether or not the hot Solar corona can possibly be hydrostatic (like the isothermal, thin, exponential atmosphere is).

We will write down the balance between pressure and gravity (HSEQ), and calculate the run of pressure versus radius, again for the isothermal case (note that pressure and density are proportional for an isothermal ideal gas and in this problem, it will be more instructive to solve for the pressure rather than the density). If you want, you can write the ideal gas law in terms of the sound speed as $p = \rho a^2/\gamma$ where the sound speed is given by $a = \sqrt{\frac{\gamma k T}{\mu m_{\rm p}}}$ and is of order the mean thermal speed of the average particle[1]. For an isothermal wind we can simply take $\gamma = 1$.

(a) What is the sound speed in the Solar photosphere ($T = 5800$ K)? What about a hot star's photosphere ($T = 50,000$ K)? What about in the Solar corona ($T = 2 \times 10^6$ K)? How far from the Sun's surface does the sound speed of the corona exceed the *local* escape velocity of the Sun?

(b) After writing down HSEQ (but in terms not of g but of GM/r^2), integrate the equation to find a solution for $p(r)$. Apply an inner boundary condition that $\rho = \rho_{\rm o} = 10^{-11}$ kg m^{-3} = 10^{-14} g cm^{-3} at the base of the corona ($r = R_{\rm o} = 7.0 \times 10^5$ km) and an outer boundary condition of arbitrary radius, r, to find an expression for the pressure of the corona at that distance. First write down a general solution and then plug

[1] It is probably more accurate to say that the actual sound speed involves only the thermal speed of the dominant species; putting the μ in the sound speed expression defines a kind of mean sound speed for a mixture of particle types.

in the numbers. Express your solution in terms of the scale height, $H_o \equiv \frac{a_o^2}{GMR_o^{-2}}$ and assume a coronal temperature of $T = 2 \times 10^6$ K.

Before you plug in your numbers, you should be able to find a nice compact expression for the pressure in terms of H_o, R_o, and p_o – the pressure of the corona at the base, $p(r = R_o)$.

(c) What is the asymptotic pressure as $r \to \infty$? The pressure does *not* go to zero, which means the pressure gradient must get very, very small at large r. Why is that, physically, given the meaning of HSEQ and the particulars of this problem/situation?

(d) The asymptotic pressure must match some outer boundary condition for the HSEQ solution to exist. The outer pressure boundary condition does not necessarily have to be $p = 0$ because the interstellar medium is out there. Typical values for the ISM density and temperature are $n = 10^7$ hydrogen atoms per m^3 (or 10 per cubic cm) and $T = 8000$ K. Is it possible for the hydrostatic Solar corona to match the ISM pressure at great distances?

See you've just proven that an isothermal solar corona cannot be static. It must be expanding – the Solar wind has to exist.

Q4 Explain what P Cygni profiles are and what gives them their characteristic shapes. How can you tell the terminal velocity of the wind from measuring a particular property of an observed P Cygni profile? Also, think about how you could go from measuring the amount of absorption in a P Cygni profile to estimating the mass-loss rate of a wind – what steps/calculations/corrections would you have to apply?

Prove mathematically that a geometrically thin spherical shell in which all material is moving at the same speed, radially away from the star (see Fig. 2.5), generates an emission line profile that is flat-topped (i.e. rectangular in shape, or the same intensity at each wavelength – though zero intensity beyond the most extreme red- and blue-shifts observed from the shell). Ignore absorption of the emitted photons and any occultation by the star. Assume that the emissivity of the wind material in the shell is proportional to the density, which is constant in the shell. Get in touch if you want a hint or two...I'd recommend considering the shell in spherical polar coordinates with the z-axis pointing at the observer, and then writing down an expression for the luminosity of the shell; then recognizing that the spectrum – the line profile – is the luminosity per unit wavelength interval, find a way to write $\frac{dL}{d\lambda}$ that takes advantage of the fact that a given Doppler shift comes from all the parts of the shell that are at a particular value of the coordinate variable θ.

The radiation-driving of stellar winds is a process that must, of course, conserve energy. How does the qualitative appearance of P Cygni line profiles provide evidence of energy conservation (in the combined stellar photospheric radiation field and wind system)? You might consider what the P Cygni profile region of the spectrum would look if there were no wind and then compare that the actual appearance and consider the total photon energy content of the spectrum in each case.

Q5 Photons have momentum, as we've seen earlier in the semester when we derived the Eddington limit (maximum luminosity a star can have without blowing itself apart). Hot star winds are driven by the transfer of momentum from photons to the wind ions via line scattering opacity (this is bound-bound absorption, followed very quickly by spontaneous emission).

If all incoming photons are from one direction but re-emission is isotropic (the same in all directions) explain how net momentum pointing radially away from the star is transferred to an ion by this process. How much momentum is transferred to an ion by one scattering event, on average? How does that compare to the momentum of an ion far out in the wind, traveling at the terminal velocity? (You can use the O star wind terminal velocity from an earlier problem and assume we're considering a carbon ion.) So...lots of scattering events per ion to accelerate the wind all the way.

Cool star winds, like the Solar wind, are *not* driven by radiation, but rather by gas pressure gradients in the Solar/stellar corona (as we have seen in Q3). But is this because their radiation fields don't contain enough momentum?

Write down an expression for the total photon momentum emitted by a star[2] in the radial direction each second, in terms of the star's luminosity, L. It should be a quite simple expression. Compute the numerical value of this quantity for the Sun and compare it to the Solar wind "momentum loss rate" (momentum per second, in the radial direction). You can assume the Solar wind has a mass-loss rate of $\dot{M} = 2 \times 10^{-14}$ $M_\odot$ yr^{-1} and a terminal velocity of $v_\infty = 700$ km s^{-1}.

Does the Sun's radiation field have enough momentum to account for the Solar wind? The Solar wind is *not* driven by radiation pressure, even though you have found that it's not because the Sun's radiation field doesn't contain enough momentum. Why might this be – that the Sun's radiation field momentum isn't well suited to drive the Solar wind? You should think about the role opacity plays in the radiation field's acceleration of particles and why the fact that the photosphere and corona of the Sun have very different temperatures makes for a radiation-opacity "mismatch".

Do the same calculation (momentum loss rate in radiation field and in the wind) for an O5 supergiant. (Assume a mass-loss rate of $\dot{M} = 5 \times 10^{-6}$ $M_\odot$ yr^{-1} and a terminal velocity of $v_\infty = 3000$ km s^{-1}.) The temperature of an O star wind is about the same as its photosphere, making the wind's opacity well suited to absorb the momentum of the star's photospheric radiation field.

For some O stars with optically thick winds, the momentum rate in the wind can exceed the momentum rate in the radiation field. How is that possible?

Q6 For an optically thin, radiation-driven wind, the radiation force goes exactly as r^{-2} (which is just the inverse square law). For an optically thicker wind, would the radiation force likely fall off slower, the same, or faster? Would the velocity law β parameter likely then be lower, the same, or higher than in the optically thin case?

Derive an expression for the velocity law, $v(r)$, for an optically thin, radiation-driven wind, and express your result in the form of a "β velocity law" – what value of β do you find? You can simply express the net force on the wind as $F = kr^{-2}$, where k is a constant related to the strength of the radiation force and the strength of gravity. And you'll want to express the acceleration as $a = v\frac{dv}{dr}$, which is just the chain rule. And very handy! You should end up with a first-order, separable differential equation which you can integrate. Here you can use the simple boundary condition $v = 0$ at $r = R_{star}$.

Note that this scenario seems to require a super-Eddington luminosity. But that's not the case if the wind opacity is much greater than the photosphere's opacity. But the wind is made of the same material as the photosphere, so how could that happen? The answer is the Doppler shift caused by the wind's own velocity. So I ask you – under what circumstances does the total strength (integrated area, or equivalent width) of an absorption line increase when its broadening (its width) increases? You might look at any absorption line (including the absorption component of the P Cygni profile diagram or observed P Cygni profile absorption component from a real star) and ask yourself the opposite question – if you made it much narrower, could

[2] Obviously, vectorially, all the momentum of the emitted photons cancels and the star isn't being pushed by this loss of momentum, but magnitude of the momentum in the wind is a well-defined quantity, as is the rate at which this momentum is being injected into the wind.

you always make it correspondingly deeper to enable the total area, or total amount of absorption, to stay the same?

Q7 Take a look at the "Hubble expansion" of the wind as seen from the point of view of an atom in the wind flow (p. 23). Assume that the wavelength of the transition in a wind atom that absorbs a photon is given by λ_o. If that photon was emitted by the photosphere and absorbed by the wind atom, does the photon in the rest frame of the photosphere have to have a longer (redder) wavelength or a shorter (bluer) wavelength than λ_o? Think about what the star looks like from the rest frame of the wind atom and think about the Doppler shift. Now, what about a photon not directly emitted by the photosphere but rather emitted (or really scattered) by another atom, somewhere else in the wind?

If you trace the journey of a single photon from the star's surface as it repeatedly is absorbed and reemitted by atoms in the wind, what happens to its wavelength, color, and energy?

If you think of a wind atom's transition as having a line profile shape (perhaps Gaussian, due to thermal broadening), and you think of it as absorbing a photon that was emitted by the same atomic transition by an identical atom elsewhere in the wind (or the photosphere), is the atom more likely to absorb the photon on one particular side of its line profile?

If we are only considering photons related to one specific transition (as we have so far for this problem), then will a given atom in the wind be able to absorb photons that were emitted from any other arbitrary place in the wind? If not, how can we define a zone around a given atom such that nearly all of the photons that that atom can absorb will arise from within that zone? Recall that we can characterize thermal broadening by a line width that corresponds to a spread of velocities.