Topics: Stellar winds part 2

Reading:

- Review the Lamers and Cassinelli reading from last week as-needed.
- Read the first four pages of Owocki's *Radiation Driven Stellar Winds from Hot Stars* and skim page 5.

I emailed a pdf of the reading over the weekend and will also put the pdf of the website.

Summary of work to be produced:

- Hand in your re-writes of Qs 2, 3, and 4 from last week by 5pm on Wednesday.
- Bring *notes* on Owocki to seminar with you but I won't collect anything.

Scope:

We'll go over questions 4, 5, 6, and 7 from last time and then discuss Sobolev theory and line-driven winds.

Questions etc.:

Please note: I'm repeating last week's questions 4 through 7 here, appended at the bottom, for your convenience. They are unchanged from last week. The new stuff is what comes first and they are questions for you to answer in order to enable our discussion of the line-driven winds article. You do not have to hand in written answers but to actively participate – and learn the material – you will want to have not just thought about these but actually write some things down.

Related to the Owocki article:

For a wind with a beta-velocity law with $v_\infty = 3000$ km s^{-1} and $\beta = 1$ and a thermal velocity of 20 km s^{-1}, what is the Sobolev length at $r = 1.2$ R$_*$? At $r = 3$ R$_*$?

For this same wind, at what radius, r, is a photon absorbed if it has a wavelength of $\lambda = 1490$ Å in the rest frame of the star and interacts with an atom in the wind via a line transition that has a rest wavelength of $\lambda = 1500$ Å?

Verify that Owocki's equation (2) leads to equations (3) and (4) in the appropriate limits.

Note that equation (3) is basically the "Eddington force" but scaled by the ratio of bound electron opacity to free electron opacity. Note further that equation (4) looks nothing like equation (3). Do you see why the optically thick line force should scale like the velocity gradient? (Think about the Sobolev approximation.)

Use equation (5) and the Sobolev approximation as embodied in equation (6) to derive equation (2). See how the locality of spectral line physics inherent in the Sobolev approximation provides numerous benefits (e.g. the avoidance of nasty spatial integrals)?

From last time:

Q4 Explain what P Cygni profiles are and what gives them their characteristic shapes. How can you tell the terminal velocity of the wind from measuring a particular property of an observed P Cygni profile? Also, think about how you could go from measuring the amount of absorption in a P Cygni profile to estimating the mass-loss rate of a wind – what steps/calculations/corrections would you have to apply?

Prove mathematically that a geometrically thin spherical shell in which all material is moving at the same speed, radially away from the star (see Fig. 2.5), generates an emission line profile that is flat-topped (i.e.

rectangular in shape, or the same intensity at each wavelength – though zero intensity beyond the most extreme red- and blue-shifts observed from the shell). Ignore absorption of the emitted photons and any occultation by the star. Assume that the emissivity of the wind material in the shell is proportional to the density, which is constant in the shell. Get in touch if you want a hint or two...I'd recommend considering the shell in spherical polar coordinates with the z-axis pointing at the observer, and then writing down an expression for the luminosity of the shell; then recognizing that the spectrum – the line profile – is the luminosity per unit wavelength interval, find a way to write $\frac{dL}{d\lambda}$ that takes advantage of the fact that a given Doppler shift comes from all the parts of the shell that are at a particular value of the coordinate variable θ.

The radiation-driving of stellar winds is a process that must, of course, conserve energy. How does the qualitative appearance of P Cygni line profiles provide evidence of energy conservation (in the combined stellar photospheric radiation field and wind system)? You might consider what the P Cygni profile region of the spectrum would look if there were no wind and then compare that the actual appearance and consider the total photon energy content of the spectrum in each case.

Q5 Photons have momentum, as we've seen earlier in the semester when we derived the Eddington limit (maximum luminosity a star can have without blowing itself apart). Hot star winds are driven by the transfer of momentum from photons to the wind ions via line scattering opacity (this is bound-bound absorption, followed very quickly by spontaneous emission).

If all incoming photons are from one direction but re-emission is isotropic (the same in all directions) explain how net momentum pointing radially away from the star is transferred to an ion by this process. How much momentum is transferred to an ion by one scattering event, on average? How does that compare to the momentum of an ion far out in the wind, traveling at the terminal velocity? (You can use the O star wind terminal velocity from an earlier problem and assume we're considering a carbon ion.) So...lots of scattering events per ion to accelerate the wind all the way.

Cool star winds, like the Solar wind, are *not* driven by radiation, but rather by gas pressure gradients in the Solar/stellar corona (as we have seen in Q3). But is this because their radiation fields don't contain enough momentum?

Write down an expression for the total photon momentum emitted by a star[1] in the radial direction each second, in terms of the star's luminosity, L. It should be a quite simple expression. Compute the numerical value of this quantity for the Sun and compare it to the Solar wind "momentum loss rate" (momentum per second, in the radial direction). You can assume the Solar wind has a mass-loss rate of $\dot{M} = 2 \times 10^{-14}$ $M_\odot$ yr^{-1} and a terminal velocity of $v_\infty = 700$ km s^{-1}.

Does the Sun's radiation field have enough momentum to account for the Solar wind? The Solar wind is *not* driven by radiation pressure, even though you have found that it's not because the Sun's radiation field doesn't contain enough momentum. Why might this be – that the Sun's radiation field momentum isn't well suited to drive the Solar wind? You should think about the role opacity plays in the radiation field's acceleration of particles and why the fact that the photosphere and corona of the Sun have very different temperatures makes for a radiation-opacity "mismatch".

Do the same calculation (momentum loss rate in radiation field and in the wind) for an O5 supergiant. (Assume a mass-loss rate of $\dot{M} = 5 \times 10^{-6}$ $M_\odot$ yr^{-1} and a terminal velocity of $v_\infty = 3000$ km s^{-1}.) The

[1]Obviously, vectorially, all the momentum of the emitted photons cancels and the star isn't being pushed by this loss of momentum, but magnitude of the momentum in the wind is a well-defined quantity, as is the rate at which this momentum is being injected into the wind.

temperature of an O star wind is about the same as its photosphere, making the wind's opacity well suited to absorb the momentum of the star's photospheric radiation field.

For some O stars with optically thick winds, the momentum rate in the wind can exceed the momentum rate in the radiation field. How is that possible?

Q6 For an optically thin, radiation-driven wind, the radiation force goes exactly as r^{-2} (which is just the inverse square law). For an optically thicker wind, would the radiation force likely fall off slower, the same, or faster? Would the velocity law β parameter likely then be lower, the same, or higher than in the optically thin case?

Derive an expression for the velocity law, $v(r)$, for an optically thin, radiation-driven wind, and express your result in the form of a "β velocity law" – what value of β do you find? You can simply express the net force on the wind as $F = kr^{-2}$, where k is a constant related to the strength of the radiation force and the strength of gravity. And you'll want to express the acceleration as $a = v\frac{dv}{dr}$, which is just the chain rule. And very handy! You should end up with a first-order, separable differential equation which you can integrate. Here you can use the simple boundary condition $v = 0$ at $r = R_{star}$.

Note that this scenario seems to require a super-Eddington luminosity. But that's not the case if the wind opacity is much greater than the photosphere's opacity. But the wind is made of the same material as the photosphere, so how could that happen? The answer is the Doppler shift caused by the wind's own velocity. So I ask you – under what circumstances does the total strength (integrated area, or equivalent width) of an absorption line increase when its broadening (its width) increases? You might look at any absorption line (including the absorption component of the P Cygni profile diagram or observed P Cygni profile absorption component from a real star) and ask yourself the opposite question – if you made it much narrower, could you always make it correspondingly deeper to enable the total area, or total amount of absorption, to stay the same?

Q7 Take a look at the "Hubble expansion" of the wind as seen from the point of view of an atom in the wind flow (p. 23). Assume that the wavelength of the transition in a wind atom that absorbs a photon is given by λ_o. If that photon was emitted by the photosphere and absorbed by the wind atom, does the photon in the rest frame of the photosphere have to have a longer (redder) wavelength or a shorter (bluer) wavelength than λ_o? Think about what the star looks like from the rest frame of the wind atom and think about the Doppler shift. Now, what about a photon not directly emitted by the photosphere but rather emitted (or really scattered) by another atom, somewhere else in the wind?

If you trace the journey of a single photon from the star's surface as it repeatedly is absorbed and reemitted by atoms in the wind, what happens to its wavelength, color, and energy?

If you think of a wind atom's transition as having a line profile shape (perhaps Gaussian, due to thermal broadening), and you think of it as absorbing a photon that was emitted by the same atomic transition by an identical atom elsewhere in the wind (or the photosphere), is the atom more likely to absorb the photon on one particular side of its line profile?

If we are only considering photons related to one specific transition (as we have so far for this problem), then will a given atom in the wind be able to absorb photons that were emitted from any other arbitrary place in the wind? If not, how can we define a zone around a given atom such that nearly all of the photons that that atom can absorb will arise from within that zone? Recall that we can characterize thermal broadening by a line width that corresponds to a spread of velocities.