Topics: Radiation transport, part 2

Reading:

- Review last week's reading in LeBlanc, Ch. 3, especially the latter part – starting on p. 77.
- When you get to the end of last week's reading (top p. 84) keep going to the end of the chapter. But you can skim (not skip!) sections 10 and 12.2.
- Re-read your solutions to the final three problems from last week's assignment.

Summary of work to be produced:

- Hand in your solutions to the warm-up questions (QW1, QW3, and QW4) by Thursday at 12:30 pm in the box on the wall outside my office.
- Bring solutions to seminar on Friday for all the (non-warm-up) numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope: Once we've got the formal solution...it's all about applying it in various situations. Well, actually, there are other things too. As I noted at the board last week, approximations of the specific intensity of the form $a + bu$ or $a + b\tau$ may seem like "toy" models but they are physically sensible inside stars.

Questions etc.:

Remember: your written answers are required only for numbered, bold-faced questions and the extent of each question is just the paragraph with the bold-face label. There are unnumbered questions between some numbered questions. Come prepared to answer/discuss these, but no need to hand anything in.

We'll go over problems 3.2, 3.3, and 3.4 (Qs 6, 7, and 8 from last week) early in the seminar. But first, we'll go through the formal solution of the radiation transfer equation.

Review Fig. 3.8 and the meaning of the transfer equation (3.32, but elsewhere too); rereading sec. 11 should help. Which terms in the transfer equation are local properties of the matter? And which terms depend on properties of the radiation field (that arises some distance away) or are material properties but not just locally, at a given point in the star under consideration?

Make sure you're comfortable with how the transport equation is solved and how to think about the solution (3.41). *I will ask someone to go to the board and go through the solution (taking us from eq. 3.22 directly to 3.32 and 3.34, referencing Fig. 3.10., and then taking us step by step from the beginning of §3.6 to eq. 3.41).*

For a star that's infinitely optically thick, the solution is evaluated a certain way (pp. 84 and beyond). But what about a star shining through an interstellar cloud that's only modestly optically thick? How would you solve eq. 3.41 then and how would you interpret the result?

QW1 Last week you computed the mean intensity and flux inside a uniform radiation field and then also for a hemisphere-filling but otherwise uniform radiation field with specific intensity I_o. Now, please compute the *radiation pressure* in these two cases. You can use a relationship between $P_{\rm rad}$ and K from §3.12.2 along with equations 3.14 and 3.17. Use the substitution $u = cos\theta$ and do the integrals by hand. Comment on the results given the odd-function/even-function trend revealed by the mean intensity and flux calculations.

If radiation is coming from one direction, then the radiation pressure leads to a force (recall, pressure *gradients* are related to forces, not pressure itself).

Q2 Consider an atom with a cross section σ (cm^{-2} particle^{-1}) and mass, m, some distance, r, from the center of a star with luminosity, L. Derive an expression for the radiation force on the atom, recalling that the momentum of a photon is its energy divided by its speed and that the force is the rate of change of momentum. Hint: What we call flux is the flux of *energy* (ergs s^{-1} cm^{-2}), but you could also modify the inverse square law to write down an expression for the flux of momentum at a given distance from a star of luminosity, L. Then you just need to think about how the momentum flux flowing past the location of the atom is related to the momentum per second being absorbed by the atom. As the final part of this question, go from the radiation force on the atom to the acceleration of that atom due to the radiation force. Can you express the acceleration in terms of opacity, k, rather than cross section?

QW3 Consider a location inside a star. The intensity of light is a function of direction (u); but let's approximate this variation as follows: for the hemisphere that's toward the surface of the star, assume that all the inward directed radiation has intensity $I_{\rm in}$ and for the hemisphere that's toward the star's center, assume a somewhat higher value for the intensity $I_{\rm out}$. The intensity is uniform within each hemisphere, but different between the two hemispheres. This is the "Eddington two-stream approximation" – see the relevant hand-out in the packet from last week (it's p. 282 from Ostlie and Carroll and I'll post it on the website). For this radiation field, compute the mean intensity, flux, and radiation pressure (answers should be in terms of $I_{\rm in}$ and $I_{\rm out}$). Comment on the results – how do they compare to the full solution in an isotropic field (if $I_{\rm out} = I_{\rm in}$) and the solution in the hemisphere case (so either one of the two I's is zero) from last week? Is the two-stream approximation good in these cases?

Radiative equilibrium: Flux inside a star is the same at every depth. Does that seem weird? The star is hotter the deeper you go; it emits like a blackbody pretty much everywhere – isn't a hotter blackbody brighter than a cooler one? What's going on here...can you convince yourself that it's logical that the flux is the same (in total, if not wavelength-by-wavelength) at every depth of the star (and thus that it's equal to $\sigma T_{\rm eff}^4$...at every depth)? Remember, everywhere but the surface, there's intensity going in as well as intensity going out.

QW4 Problem 3.5. Please also answer the following: What's the physical interpretation of the quantity $k\rho_{\rm o}z_{\rm o}$? And what is the average density of the cloud. You might refer to Fig. 3.10 to make sure you've got the *where is* $\tau = 0$*? in which direction does* τ *increase?* convention right.

LTE: Comfortable with $S = B$ in LTE? Are the units okay? If we can assume LTE, then the source function is just dependent on the (local) temperature. Not dependent on knowing the ionization balance and level populations; not dependent on the composition. Just the temperature.

Q5 Consider a uniform interstellar cloud as in QW4. Show that the solution to the transfer equation gives emission lines when there's no background source and the cloud is optically thin. Hint, you'll want to use the Taylor expansion of the exponential attenuation, which is justified (using just the first two terms) in the optically thin case. And further, you can assume that spectral lines exist at wavelengths or frequencies where the opacity is *high* compared to the continuum. Realize that since the cloud is a discrete object, not part of a star, the optical depth of the cloud goes from $t = 0$ at the front to $t = \tau$ at the back. The front of the cloud is the side radiation comes out of (so it corresponds to τ_1 in equation 3.41 and Fig. 3.11). Finally, see if you can express your final answer for I as a function of the emissivity, j, rather than the opacity, k, and then interpret the result as showing emission lines based on the physical meaning of j.

We'll go over **questions 6, 7, and 8** from last week at about this point in the seminar. Be prepared to interpret the results and the relevance of the questions and the significance of their solutions.

We you read §3.8 and eq. 3.73 think about how it could describe heat flow out of a poorly insulated house on a cold day and think about parallels to Ohm's law. In §3.9 be prepared to discuss specific opacity sources mentioned in the section and why they might have particular temperature dependencies; and for §3.12.1 be prepared to show that eq. 3.103 has units of acceleration.

Q6 Problem 3.6.

Q7 Problem 3.7.

Q8 Problem 3.8. Consider making a change of variables to $Y = I - S$.