Topics: Stellar atmospheres, part 2: including spectral formation and diagnostics

Reading:

- LeBlanc, Ch. 4 (the second half, from p. 132 through p. 151).
- (From last week:) A few pages (pp. 276-78) from Ostlie and Carroll's textbook: on the diffusion (random walk) of radiation (pdf available on the website).
- We may add a small amount of outside reading (and/or actual stellar spectra to study).

Summary of work to be produced:

- For now, no warmup problems – but a couple may be designated as warm-ups.
- Bring solutions to seminar on Friday for all the (non-warm-up) numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope: Understanding the physics of the formation of spectral lines enables us to derive information about stars and their atmospheres via measurements of their spectra. We'll have to revisit atomic processes and Einstein co-efficients and line profile functions, among other things.

Questions etc.:

Q0 (Left over from last time) Read the derivation of the random walk of a photon in Ostlie and Carroll and then answer the following question: According to the standard model of the Sun, the central density is 162 g cm^{-3} and the Rosseland mean opacity at the center is 1.16 cm^2 g^{-1}. **(a)** Calculate the mean free path of a photon at the Sun's center. **(b)** If the mean free path remains constant for the photon's journey to the surface of the Sun, calculate the average time it would take for the photon to escape from the Sun. **(c)** Using the plot of the Rosseland mean opacity in Ch. 3 and the knowledge that the density at the Sun's surface is approximately 10^{-8} g cm^{-3}, is your answer to part (b) likely to be an overestimate or an underestimate for the actual Sun? What's your reasoning for this last part? Note that you should be able to demonstrate the derivation at the board. Are you okay with the dot product? And the cancellation of cross-terms in the big sum?

Why is stimulated emission treated as "negative opacity" rather than like an analog of spontaneous emission?

Is stimulated emission more important for long or short wavelengths?

Note the discussion of (artificial) lasers. Why do the level populations in a laser have to be non-thermal? Can the Boltzmann distribution ever give $n_{\rm upper} >> n_{\rm lower}$? Note that the book uses subscripts i and j for the lower and upper levels.

Q1 Problem 4.10.

What are the three Einstein coefficients...in words? What are their units?

What does eq. 4.78 say/mean? If it holds (note it describes a simplified two-level atom in this form) are the level populations changing with time?

Aside: Einstein argued 4.78 on thermodynamic grounds and 4.83 and 4.84 based on quantum mechanical principles and from there, he *derived* the Planck function after positing that stimulated emission must exist.

Note that B_{ij} is related to the oscillator strength (f). Remind yourself about the oscillator strength and r.h.s. of eq. 4.85 (what is $\frac{\pi e}{m_e c} f$?).

Q2 Problem 4.6. You'll want to incorporate the Boltzmann equation into your solution. (And so you're assuming LTE.)

See how the Einstein A coefficient governs natural broadening? Remind yourself of the physical casue of natural broadening and argue that 4.87 makes sense (how is A related to the lifetime of an excited state?).

Think carefully about the definition of equivalent width – both conceptual and mathematical. Does it really have to do with the *width* of a line? What are its units? What is A_λ and what are its units?

Q3 Problem 4.9.

What defines a "weak" line? Why is "optically thin" or "optically thick" more complicated for a stellar atmosphere than for an interstellar cloud with a light source behind it?

What does the temperature gradient in the atmosphere have to do with the existence of absorption lines?

Why/how does eq. 4.94 make sense? What assumptions go into it?

What's going on in Fig. 4.16 (and the text below it)? Why (conceptually) is $\Delta\tau < 0$?

Note the Taylor expansion of $B_\lambda(\tau = 2/3)$ and the derivation of the expression for A_λ in terms of k_ℓ and the temperature gradient (or really $\frac{dB}{d\tau}$). Is it reasonable that the opacity is proportional to the abundance of the element that gives rise to the line in question? For a particular line, is k_ℓ also proportional to temperature, density, or anything else? In what context can measuring the equivalent widths of weak lines give us unambiguous information about the elemental abundance? Why does the technique break down for strong lines?

Study Fig. 4.17. Why do the line depths and shapes change in the ways they do as the abundance is increased? Why doesn't the very optically thick line go to zero flux?

For the curve of growth, it's stated that weak lines have equivalent widths that are linearly proportional to abundance. Closely related: show that the depth of an optically thin absorption line in a cloud back-lit by a light source is proportional to the cloud's column density.

What is the cause of the two more optically thick regimes in the curve of growth? You should be thinking about Gaussian vs. Lorentzian line shape properties – and the properties of their respective wings, perhaps.

The Atmosphere Modelling section starts to tie things together...

Gravity and effective temperature are the usual parameters that define a given model atmosphere. Does it makes sense that in the plane parallel approximation, the radius of the star doesn't matter at all?

Take notice of what kinds/aspects of atomic data are required.

Q4 Problem 4.13. Note that you can use one of the results of the grey atmosphere. And you can assume that the temperature at which ionization occurs is linearly proportional to the ionization potential.

Why does opacity depend on temperature?

Note that the frequency grid for the structure calculation can be coarse, and then once the atmosphere structure (what's meant by this?) is solved, a much finer frequency grid can be used to synthesize a spectrum.

The equation of state is mentioned near the end of the section. Why does the pressure (at each layer) need to be known/computed?

The algorithm for atmosphere modelling – study Fig. 4.19. Does it make sense that the procedure has to be iterative? What quantities/equations depend on each other?

Inputs are $T_{\rm eff}$ and g, abundances, and atomic data. And we require that the flux is the same at all levels.

Aside: the "more realistic" treatment of the Saha equation – note that adding more elements makes things a lot more complicated (to some extent because we need to know the free electron density to solve the Saha equation for any pair of ionization states).

Q5 Problem 4.11.

Note why having HSEQ in terms of log τ is useful.

Note "...various astrophysical applications..." includes measuring abundances using all lines, not just weak ones.

Looking at the top two panels of Fig. 4.20, roughly what is the column mass above $\tau = 1$ (in cgs)? And looking at the bottom panel, what is the pressure of the Earth's atmosphere at sea level in these units? What about at the center of the Sun (assuming a perfect gas and a density 140 times that of water and a temperature of 15.7×10^6 K)? What temperature blackbody would emit light that has a radiation pressure comparable to the actual pressure of this atmosphere at $\tau \approx 2/3$?

Think about the last paragraph of §4.5.3, where the effect of H ionization on some of the properties of the curves is discussed. Why should ionization matter?

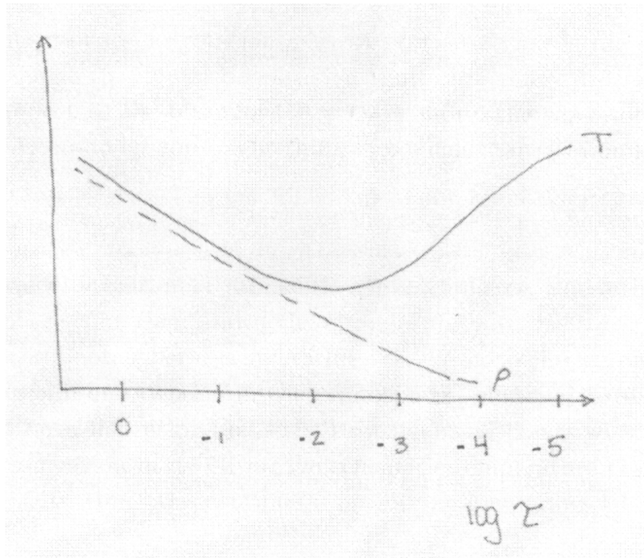
Read about the temperature correction procedure – remind yourself what radiative equilibrium (beyond just the grey atmosphere) is.

Q7 Problem 4.12.

Q8 For a random distribution of stellar rotation axis orientations, what is the probability of finding a star with any given inclination angle (p. 129 for definition)? Hint: How many distinct orientations in 3-D space give $i = 0$ degrees with respect to the observer? Are there more, the same, or fewer distinct orientations that give $i = 90$ degrees? Alternately, you might think of a single star surrounded by observers on the unit sphere.

Q9 Problem 4.14.

Q10 Stars like the Sun have a chromosphere above their photospheres, where the temperature is somewhat lower than in the photosphere, but then above the chromosphere is the *transition region* where the temperature rises, exceeding the photospheric temperature, as the hot solar corona is approached. Below is a schematic of the temperature and density in the photosphere, chromosphere, and transition region as a function of the continuum optical depth. The so-called calcium K line is so strong that you can actually see evidence of the transition region in it. Given the schematic temperature gradient, sketch the Ca K line assuming that its opacity is 10^4 times the continuum opacity and assuming LTE. You can assume that the line has a Lorentzian shape.


T
ρ
0
-1
-2
-3
-4
-5
log τ