Topics: Stellar interiors, part 1

Reading:

- LeBlanc, Ch. 5 (the first half, from p. 155 through p. 172.
- Optionally, to supplement LeBlanc, you can read in Ch. 10 of Ostlie and Carroll's *An Introduction to Modern Stellar Astrophysics*, which I've requested to have put on reserve in Cornell.
- I will also make some thermodynamics notes available to refresh our memories so we can understand better the polytropic index, γ, that shows up in §3. Both this and the previous resource will be useful next week as well.

Summary of work to be produced:

- There are two warmup problems due on Thursday at 6 pm: QW1 and QW2.
- Bring solutions to seminar on Friday for all the (non-warm-up) numbered problems. Bring a xeroxed copy to give to me at the beginning of class, and expect to take notes on your original solutions.

Scope: We go down, below the surfaces of stars this week! You might want to review your Astro 16 notes and Ch. 15 from Ryden and Peterson (Astro 14/16 book), especially if you find anything in the LeBlanc reading to be confusing or you just want a different perspective. Plus, see the optional/supplementary readings listed above.

Stellar structure is described by four differential equations. Plus opacities, energy generation equations that describe nuclear reactions, abundances (including their possible stratification – differences in the abundance of a given element as a function of radius within the star), equation of state (pressure-density-temperature relations - though full equations of state will wait until the following week), and boundary conditions.

Then...we also need to recognize the energy may be transported not just by radiation but also by convection and even conduction.

We'll start next week with polytropic models – structure models that make some thermodynamic assumptions and thus enable models to be build without knowledge of the energy sources inside stars and indeed without explicit solving of the temperature structure within stars.

Questions etc.:

Hydrostatic equilibrium (eq. 5.1) is an equilibrium condition in the sense that there's no motion (the *static* part). Specifically how does eq. 5.2, which relaxes the equilibrium assumption, imply *motion*?

Along similar lines, take a look at the mass continuity equation (eq. 5.4), and multiply both sides by a velocity and find an equation that describes a more relaxed equilibrium (steady-state) assumption and incorporates motion.

QW1 Problem 5.1

LeBlanc derives the radiation transport equation (eq. 5.18) using "theoretical developments" from chapter 3. **QP 1** Someone will give a presentation on the derivation of eq. 5.18 comparing LeBlanc's method and the method that Ryden and Peterson use in *Foundations of Astrophysics* (Astro 14/16 textbook), in §15.1.2 on pp. 353-54.

QW2 Problem 5.2 – you can start with eq. 5.18

Q3 Problem 5.3

Q4 Problem 5.4

Q5 Problem 5.5

Q6 Problem 5.6

Q7 Problem 5.7

Q8 Problem 5.8

Q9 Convection:

Let's derive the criterion that determines whether convection transports energy at a given location in a star.

Consider two cylindrical parcels of material (denoted *prime* and *unprime*), each of the same height dr and surface area dA and at the same radial location within a star. The unprimed parcel (with properties T, ρ, etc.) is in hydrostatic equilibrium, and can be considered the "average" stellar material. The primed parcel (with properties T', ρ', etc.) may not be in equilibrium and is like a bubble that might rise, sink, or remain stationary. Both parcels are at the same location in the star, so they are surrounded by the same material (which itself has properties identical to the unprimed parcel). Note that all the physical properties (temperature, density, pressure, even the mean molecular weight) are functions of r, location in the star – the radial coordinate, or distance from the center. If a parcel rises or falls, its own properties may change and the properties of the surrounding material may change, and they may change *in different ways*.

(a) Use Newton's second law to derive an equation for the forces acting on the unprimed parcel, which is at rest. Your expression should start out being in terms of $P_{\rm top}$ and $P_{\rm bottom}$, the pressures on the top and bottom of the parcel, respectively; and the density, ρ; the surface area on top or bottom, dA; on $M_{\rm r}$, the mass interior to the parcel's radial location, r; the gravitational constant, G; and r. Carefully label forces on the parcels in a diagram and make sure all the negative signs are in the right places in your equation. Looking at the derivation of HSEQ might be helpful if you're having trouble getting started, or perhaps just thinking about the very first unnumbered question in this assignment would help.

(b) Use Newton's second law to derive an equation for the forces acting on the *primed* parcel (the "bubble"), which may have a non-zero acceleration.

(c) Combine (a) and (b) to find an expression for the acceleration in terms of G, $M_{\rm r}$, ρ, ρ', and r. Simplify your expression as much as possible.

Under what conditions will the primed parcel rise, sink, or remain in place?

(d) Convert your condition in part (c) to a condition on the *temperature* in the unprimed and primed parcels (assume the perfect gas law).

What does this condition imply about the *direction* of energy transfer when convection occurs?

(e) So far, you have only determined the conditions under which the primed parcel will *instantaneously* accelerate up or down or remain fixed in place. But the more important question is the following: Presume that the primed parcel is indeed perturbed so as to instantaneously accelerate upwards, in accord with part (c). Once above its original location, will the primed parcel continue to accelerate upwards or will it stop and sink back toward its equilibrium position?

Clearly, as the bubble rises, the density and pressure surrounding it will decrease. The density of the bubble will also change as it rises. However, they need not vary at the same rate. Show that if the radial density gradient of the bubble is steeper than the radial density gradient of its surroundings, then once the bubble begins to rise in accord with the condition in part (c), it will continuously accelerate upward.

Convection is an efficient energy transfer mechanism in this case.

Conversely, if the radial density gradient of the surroundings is steeper than the density gradient of the bubble, show that the bubble will begin an upward journey if condition (c) is satisfied, but it will soon stop.

In this case, energy cannot be transported by convection.

The best way to solve this final part of the problem is via a sketch of the density vs. radius for the two parcels, along with some words describing what's going on and referring to the solution to (c). This sketch shouldn't be quantitative, but it should show if the two variables increase or decrease with radius, and which one has the steeper slope.

(f) What if the parcel has a different composition (but the same temperature) as its surroundings. (How) could this drive convection?

Can you explain how what you've just shown corresponds to eq. 5.45 and subsequent equations?