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PROBLEMS

16.1 The most easily observed white dwarf in the sky is in the constellation of Eridanus (the River Eridanus). Three stars make up the 40 Eridani system: 40 Eri A is a 4th-magnitude star similar to the Sun; 40 Eri B is a 10th-magnitude white dwarf; and 40 Eri C is an 11th-magnitude red M5 star. This problem deals only with the latter two stars, which are separated from 40 Eri A by 400 AU.

(a) The period of the 40 Eri B and C system is 247.9 years. The system's measured trigonometric parallax is 0.201″ and the true angular extent of the semimajor axis of the reduced mass is 6.89″. The ratio of the distances of 40 Eri B and C from the center of mass is $a_B/a_C = 0.37$. Find the mass of 40 Eri B and C in terms of the mass of the Sun.

(b) The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in terms of the luminosity of the Sun.

(c) The effective temperature of 40 Eri B is 16,900 K. Calculate its radius, and compare your answer to the radii of the Sun, Earth, and Sirius B.

(d) Calculate the average density of 40 Eri B, and compare your result with the average density of Sirius B. Which is more dense, and why?

(e) Calculate the product of the mass and volume of both 40 Eri B and Sirius B. Is there a departure from the mass–volume relation? What might be the cause?

16.2 The helium absorption lines seen in the spectra of DB white dwarfs are formed by excited He I atoms with one electron in the lowest ($n = 1$) orbital and the other in an $n = 2$ orbital. White dwarfs of spectral type DB are not observed with temperatures below about 11,000 K. Using what you know about spectral line formation, give a *qualitative* explanation why the helium lines would not be seen at lower temperatures. As a DB white dwarf cools below 12,000 K, into what spectral type does it change?

16.3 Deduce a rough upper limit for X, the mass fraction of hydrogen, in the interior of a white dwarf. *Hint:* Use the mass and average density for Sirius B in the equations for the nuclear energy generation rate, and take $T = 10^7$ K for the central temperature. Set ψ_{pp} and $f_{pp} = 1$ in Eq. (10.47) for the pp chain, and $X_{\rm CNO} = 1$ in Eq. (10.59) for the CNO cycle.

16.4 Estimate the ideal gas pressure and the radiation pressure at the center of Sirius B, using 3×10^7 K for the central temperature. Compare these values with the estimated central pressure, Eq. (16.1).

16.5 By equating the pressure of an ideal gas of electrons to the pressure of a degenerate electron gas, determine a condition for the electrons to be degenerate, and compare it with the condition of Eq. (16.6). Use the exact expression (Eq. 16.12) for the electron degeneracy pressure.

16.6 In the extreme relativistic limit, the electron speed $v = c$ must be used instead of Eq. (16.10) to find the electron degeneracy pressure. Use this to repeat the derivation of Eq. (16.11) and find

$$P \approx \frac{\hbar c}{\sqrt{3}} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{4/3}.$$

16.7 **(a)** At what speed do relativistic effects become important at a level of 10%? In other words, for what value of v does the Lorentz factor, γ, become equal to 1.1?

(b) Estimate the density of the white dwarf for which the speed of a degenerate electron is equal to the value found in part (a).

(c) Use the mass–volume relation to find the approximate mass of a white dwarf with this average density. This is roughly the mass where white dwarfs depart from the mass–volume relation.

16.8 Crystallization will occur in a cooling white dwarf when the electrostatic potential energy between neighboring nuclei, $Z^2e^2/4\pi\epsilon_0 r$, dominates the characteristic thermal energy kT.

The ratio of the two is defined to be Γ,

$$\Gamma = \frac{Z^2 e^2}{4\pi\epsilon_0 r k T}.$$

In this expression, the distance r between neighboring nuclei is customarily (and somewhat awkwardly) defined to be the radius of a sphere whose volume is equal to the volume per nucleus. Specifically, since the average volume per nucleus is Am_H/ρ, r is found from

$$\frac{4}{3}\pi r^3 = \frac{Am_H}{\rho}.$$

(a) Calculate the value of the average separation r for a 0.6 $M_\odot$ pure carbon white dwarf of radius 0.012 $R_\odot$.

(b) Much effort has been spent on precise numerical calculations of Γ to obtain increasingly realistic cooling curves. The results indicate a value of about $\Gamma = 160$ for the onset of crystallization. Estimate the interior temperature, T_c, at which this occurs.

(c) Estimate the luminosity of a pure carbon white dwarf with this interior temperature. Assume a composition like that of Example 16.5.1 for the nondegenerate envelope.

(d) For roughly how many years could the white dwarf sustain the luminosity found in part (c), using just the latent heat of kT per nucleus released upon crystallization? Compare this amount of time (when the white dwarf cools more slowly) with Fig. 16.9.

16.9 In the *liquid-drop model* of an atomic nucleus, a nucleus with mass number A has a radius of $r_0 A^{1/3}$, where $r_0 = 1.2 \times 10^{-15}$ m. Find the density of this nuclear model.

16.10 If our Moon were as dense as a neutron star, what would its diameter be?

16.11 **(a)** Consider two point masses, each having mass m, that are separated vertically by a distance of 1 cm just above the surface of a neutron star of radius R and mass M. Using Newton's law of gravity (Eq. 2.11), find an expression for the ratio of the gravitational force on the lower mass to that on the upper mass, and evaluate this expression for $R = 10$ km, $M = 1.4\ M_\odot$, and $m = 1$ g.

(b) An iron cube 1 cm on each side is held just above the surface of the neutron star described in part (a). The density of iron is 7860 kg m^{-3}. If iron experiences a stress (force per cross-sectional area) of 4.2×10^7 N m^{-2}, it will be permanently stretched; if the stress reaches 1.5×10^8 N m^{-2}, the iron will rupture. What will happen to the iron cube? (*Hint:* Imagine concentrating half of the cube's mass on each of its top and bottom surfaces.) What would happen to an iron meteoroid falling toward the surface of a neutron star?

16.12 Estimate the neutron degeneracy pressure at the center of a 1.4 $M_\odot$ neutron star (take the central density to be 1.5×10^{18} kg m^{-3}), and compare this with the estimated pressure at the center of Sirius B.

16.13 **(a)** Assume that at a density just below neutron drip, all of the neutrons are in heavy neutron-rich nuclei such as $^{118}_{36}$Kr. Estimate the pressure due to relativistic degenerate electrons.

(b) Assume (*wrongly!*) that at a density just above neutron drip, all of the neutrons are free (and not in nuclei). Estimate the speed of the degenerate neutrons and the pressure they would produce.

16.14 Suppose that the Sun were to collapse down to the size of a neutron star (10-km radius).

(a) Assuming that no mass is lost in the collapse, find the rotation period of the neutron star

(b) Find the magnetic field strength of the neutron star.

Even though our Sun will not end its life as a neutron star, this shows that the conservation of angular momentum and magnetic flux can easily produce pulsar-like rotation speeds and magnetic fields.

16.15 **(a)** Use Eq. (14.14) with $\gamma = 5/3$ to calculate the fundamental radial pulsation period for a one-zone model of a pulsating white dwarf (use the values for Sirius B) and a 1.4 $M_\odot$ neutron star. Compare these to the observed range of pulsar periods.

(b) Use Eq. (16.29) to calculate the minimum rotation period for the same stars, and compare them to the range of pulsar periods.

(c) Give an explanation for the similarity of your results.

16.16 **(a)** Determine the minimum rotation period for a 1.4 $M_\odot$ neutron star (the fastest it can spin without flying apart). For convenience, assume that the star remains spherical with a radius of 10 km.

(b) Newton studied the equatorial bulge of a homogeneous fluid body of mass M that is *slowly* rotating with angular velocity Ω. He proved that the difference between its equatorial radius (E) and its polar radius (P) is related to its average radius (R) by

$$\frac{E-P}{R} = \frac{5\Omega^2 R^3}{4GM}.$$

Use this to estimate the equatorial and polar radii for a 1.4 $M_\odot$ neutron star rotating with twice the minimum rotation period you found in part (a).

16.17 If you measured the period of PRS 1937+214 and obtained the value on page 588, about how long would you have to wait before the last digit changed from a "5" to a "6"?

16.18 Consider a pulsar that has a period P_0 and period derivative $\dot{P}_0$ at $t = 0$. Assume that the product $P\dot{P}$ remains constant for the pulsar (cf. Eq. 16.32).

(a) Integrate to obtain an expression for the pulsar's period P at time t.

(b) Imagine that you have constructed a clock that would keep time by counting the radio pulses received from this pulsar. Suppose you also have a *perfect* clock ($\dot{P} = 0$) that is initially synchronized with the pulsar clock when they both read zero. Show that when the perfect clock displays the characteristic lifetime $P_0/\dot{P}_0$, the time displayed by the pulsar clock is $(\sqrt{3} - 1)P_0/\dot{P}_0$.

16.19 During a glitch, the period of the Crab pulsar decreased by $|\Delta P| \approx 10^{-8}P$. If the increased rotation was due to an overall contraction of the neutron star, find the change in the star's radius. Assume that the pulsar is a rotating sphere of uniform density with an initial radius of 10 km.

16.20 The Geminga pulsar has a period of $P = 0.237$ s and a period derivative of $\dot{P} = 1.1 \times 10^{-14}$. Assuming that $\theta = 90°$, estimate the magnetic field strength at the pulsar's poles.

16.21 **(a)** Find the radii of the light cylinders for the Crab pulsar and for the slowest pulsar, PSR 1841-0456. Compare these values to the radius of a 1.4 $M_\odot$ neutron star.

(b) The strength of a magnetic dipole is proportional to $1/r^3$. Determine the ratio of the magnetic field strengths at the light cylinder for the Crab pulsar and for PSR 1841-0456.

16.22 **(a)** Integrate Eq. (16.32) to obtain an expression for a pulsar's period P at time t if its initial period was P_0 at time $t = 0$.

(b) Assuming that the pulsar has had time to slow down enough that $P_0 \ll P$, show that the age t of the pulsar is given approximately by

$$t = \frac{P}{2\dot{P}},$$

where $\dot{P}$ is the period derivative at time t.

(c) Evaluate this age for the case of the Crab pulsar, using the values found in Example 16.7.1. Compare your answer with the known age.

16.23 One way of qualitatively understanding the flow of charged particles into a pulsar's magnetosphere is to imagine a charged particle of mass m and charge e (the fundamental unit of charge) at the equator of the neutron star. Assume for convenience that the star's rotation carries the charge perpendicular to the pulsar's magnetic field. The moving charge experiences a magnetic Lorentz force of $F_m = evB$ and a gravitational force, F_g. Show that the ratio of these forces is

$$\frac{F_m}{F_g} = \frac{2\pi eBR}{Pmg},$$

where R is the star's radius and g is the acceleration due to gravity at the surface. Evaluate this ratio for the case of a proton at the surface of the Crab pulsar, using a magnetic field strength of 10^8 T.

16.24 Find the minimum photon energy required for the creation of an electron–positron pair via the pair-production process $\gamma \rightarrow e^- + e^+$. What is the wavelength of this photon? In what region of the electromagnetic spectrum is this wavelength found?

16.25 A subpulse involves a very narrow radio beam with a width between 1° and 3°. Use Eq. (4.43) for the headlight effect to calculate the minimum speed of the electrons responsible for a 1° subpulse.