the "bump" seen near that temperature. Finally, the plot reaches a flat floor at the right-hand side of the figure. Electron scattering dominates at the highest temperatures, when nearly all of the stellar material is ionized and there are few bound electrons available for bound–bound and bound–free processes. The form of Eq. (9.21) for electron scattering, with no density or temperature dependence, requires that all of the curves in Fig. 9.10 converge to the same constant value in the high temperature limit.

9.3 Radiative Transfer

In an equilibrium, steady-state star, there can be no change in the total energy contained within any layer of the stellar atmosphere or interior.[19] In other words, the mechanisms involved in absorbing and emitting energy must be precisely in balance throughout the star. In this section, the competition between the absorption and emission processes will be described, first in qualitative terms and later in more quantitative detail.

Any process that adds photons to a beam of light will be called **emission**. Thus emission includes the scattering of photons into the beam, as well as the true emission of photons by electrons making downward atomic transitions. Each of the four primary sources of opacity listed in Section 9.2 has an inverse emission process: bound–bound and free–bound emission, free–free emission (bremsstrahlung), and electron scattering. The processes of absorption and emission hinder the flow of photons through the star by *redirecting* the paths of the photons and *redistributing* their energy. Thus in a star there is not a direct flow of photons streaming toward the surface, carrying energy outward at the speed of light. Instead, the individual photons travel only temporarily with the beam as they are repeatedly scattered in random directions following their encounters with gas particles.

As the photons diffuse upward through the stellar material, they follow a haphazard path called a **random walk**. Figure 9.11 shows a photon that undergoes a net displacement $\mathbf{d}$ as the result of making a large number N of randomly directed steps, each of length ℓ (the mean free path):

$$\mathbf{d} = \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2 + \boldsymbol{\ell}_3 + \cdots + \boldsymbol{\ell}_N.$$

[19]This is not the case for a star that is *not* in equilibrium. In the case of pulsating stars (to be discussed in Chapter 14), a periodic absorption or "damming up" of the outward flow of energy occurs that drives the oscillations.

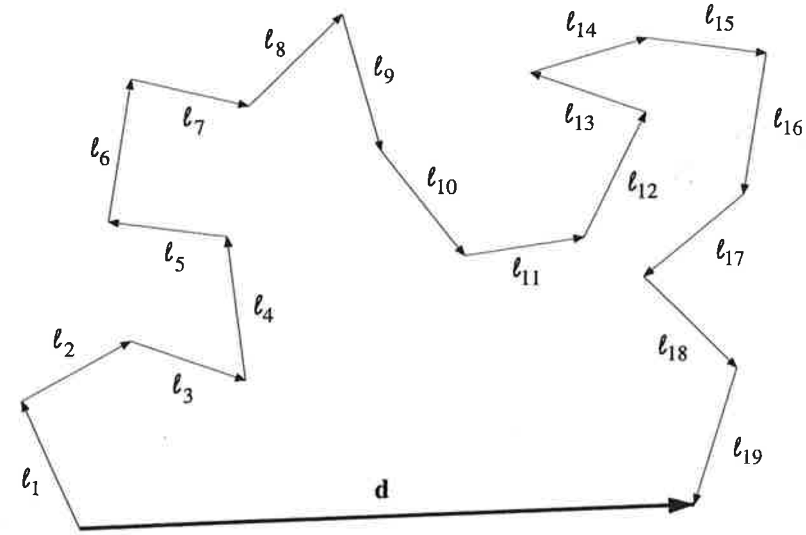

Figure 9.11 Displacement **d** of a random-walking photon.

Taking the vector dot product of **d** with itself gives

$$\begin{aligned}\mathbf{d}\cdot\mathbf{d} &= \boldsymbol{\ell}_1\cdot\boldsymbol{\ell}_1+\boldsymbol{\ell}_1\cdot\boldsymbol{\ell}_2+\cdots+\boldsymbol{\ell}_1\cdot\boldsymbol{\ell}_N\\ &\quad+\boldsymbol{\ell}_2\cdot\boldsymbol{\ell}_1+\boldsymbol{\ell}_2\cdot\boldsymbol{\ell}_2+\cdots+\boldsymbol{\ell}_2\cdot\boldsymbol{\ell}_N+\\ &\quad\cdots+\boldsymbol{\ell}_N\cdot\boldsymbol{\ell}_1+\boldsymbol{\ell}_N\cdot\boldsymbol{\ell}_2+\cdots+\boldsymbol{\ell}_N\cdot\boldsymbol{\ell}_N,\end{aligned}$$

or

$$\begin{aligned}d^2 &= N\ell^2+\ell^2[\cos\theta_{12}+\cos\theta_{13}+\cdots+\cos\theta_{1N}\\ &\quad+\cos\theta_{21}+\cos\theta_{23}+\cdots+\cos\theta_{2N}+\\ &\quad\cdots+\cos\theta_{N1}+\cos\theta_{N2}+\cdots+\cos\theta_{N(N-1)}],\end{aligned}$$

where θ_{ij} is the angle between the vectors $\boldsymbol{\ell}_i$ and $\boldsymbol{\ell}_j$. For a large number of randomly directed steps, the sum of all the cosine terms approaches zero. As a result, for a random walk, the displacement d is related to the size of each step, ℓ, by

$$d=\ell\sqrt{N}. \tag{9.23}$$

Thus the transport of energy through a star by radiation may be extremely inefficient. As a photon follows its tortuous path to the surface of a star, it takes 100 steps to travel a distance of 10ℓ; 10,000 steps to travel 100ℓ; and one million steps to travel 1000ℓ.[20] Because the optical depth at a point is

[20]As will be discussed in Section 10.4, the process of transporting energy by radiation is sometimes so inefficient that another method, *convection*, must take over.

roughly the number of photon mean free paths from that point to the surface (as measured along a light ray's straight path), Eq. (9.23) implies that the distance to the surface is $d = \tau_\lambda \ell = \ell\sqrt{N}$. The average number of steps a photon leaving the surface took to travel the distance d is then

$$N = \tau_\lambda^2, \tag{9.24}$$

for $\tau_\lambda \gg 1$. As might be expected, when $\tau_\lambda \approx 1$, a photon may escape from the surface of the star. A more careful analysis (performed later in this section) shows that the average level in the atmosphere from which photons of wavelength λ escape is at an optical depth of $\tau_\lambda = 2/3$. *Looking into a star at any angle, we always look back to an optical depth of about* $\tau_\lambda = 2/3$, *as measured straight back along the line of sight.* In fact, a star's visible surface or **photosphere** is defined as the layer from which its visible light originates, that is, where $\tau_\lambda \approx 2/3$ for wavelengths in the star's continuum.

The realization that an observer looking vertically down on the surface of a star sees photons from $\tau_\lambda \approx 2/3$ offers an important insight into the formation of spectral lines. Recalling the definition of optical depth, Eq. (9.15),

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho \, ds,$$

we see that, if the opacity κ_λ increases at some wavelength, then the actual distance back along the ray to $\tau_\lambda = 2/3$ decreases for that wavelength. One cannot see as far into murky material, so an observer will not see as deeply into the star at wavelengths where the opacity is greater than average (i.e., greater than the continuum opacity). Thus, if the temperature of the stellar atmosphere decreases outward, then these higher regions of the atmosphere will be cooler. As a result, the intensity of the radiation at $\tau_\lambda \approx 2/3$ will decline the most for those wavelengths at which the opacity is greatest, resulting in absorption lines in the continuous spectrum. Thus the temperature *must* decrease outward for the formation of absorption lines. This is the analog for stellar atmospheres of Kirchhoff's law that a cool, diffuse gas in front of a source of a continuous spectrum produces dark spectral lines in the continuous spectrum.

Another implication of looking down to an optical depth of two-thirds is shown in Fig. 9.12. The line of sight of an observer on Earth viewing the Sun is vertically down at the center of the Sun's disk but makes an increasingly large angle θ with the vertical near the edge, or *limb*, of the Sun. Looking near the limb, the observer will not see as deeply into the solar atmosphere and will therefore see a lower temperature at an optical depth of two-thirds