The Eve of a New World View

This section draws to a close at the end of the nineteenth century. The physicists and astronomers of the time believed that all of the principles that govern the physical world had finally been discovered. Their scientific world view, the *Newtonian paradigm*, was the culmination of the heroic, golden age of classical physics that had flourished for over three hundred years. The construction of this paradigm began with the brilliant observations of Galileo and the subtle insights of Newton. Its architecture was framed by Newton's laws, supported by the twin pillars of the conservation of energy and momentum and illuminated by Maxwell's electromagnetic waves. Its legacy was a deterministic description of a universe that ran like clockwork, with wheels turning inside of wheels, all of its gears perfectly meshed. Physics was in danger of becoming a victim of its own success. There were no challenges remaining. All of the great discoveries apparently had been made, and the only task remaining for men and women of science at the end of the nineteenth century was filling in the details.

However, as the twentieth century opened, it became increasingly apparent that a crisis was brewing. Physicists were frustrated by their inability to answer some of the simplest questions concerning light. What is the medium through which light waves travel the vast distances between the stars, and what is Earth's speed through this medium? What determines the continuous spectrum of blackbody radiation and the characteristic, discrete colors of tubes filled with hot glowing gases? Astronomers were tantalized by hints of a treasure of knowledge just beyond their grasp.

It took a physicist of the stature of Albert Einstein to topple the Newtonian paradigm and bring about two revolutions in physics. One transformed our ideas about space and time, and the other changed our basic concepts of matter and energy. The rigid clockwork universe of the golden era was found to be an illusion and was replaced by a random universe governed by the laws of probability and statistics. The following four lines aptly summarize the situation. The first two lines were written by the English poet Alexander Pope (1688–1744), a contemporary of Newton; the last two, by Sir J. C. Squire (1884–1958), were penned in 1926.

> Nature and Nature's laws lay hid in night:
> God said, *Let Newton be!* and all was light.
>
> It did not last: the Devil howling "Ho!
> Let Einstein be!" restored the status quo.

3.5 ■ THE QUANTIZATION OF ENERGY

One of the problems haunting physicists at the end of the nineteenth century was their inability to derive from fundamental physical principles the blackbody radiation curve depicted in Fig. 3.8. Lord Rayleigh[14] (1842–1919) had attempted to arrive at the expression by applying Maxwell's equations of classical electromagnetic theory together with the results

[14]Lord Rayleigh, as he is commonly known, was born John William Strutt but succeeded to the title of third Baron Rayleigh of Terling Place, Witham, in the county of Essex, when he was thirty years old.

from thermal physics. His strategy was to consider a cavity of temperature T filled with blackbody radiation. This may be thought of as a hot oven filled with standing waves of electromagnetic radiation. If L is the distance between the oven's walls, then the permitted wavelengths of the radiation are $\lambda = 2L, L, 2L/3, 2L/4, 2L/5, \ldots$, extending forever to increasingly shorter wavelengths.[15] According to classical physics, each of these wavelengths should receive an amount of energy equal to kT, where $k = 1.3806503 \times 10^{-23}$ J K^{-1} is Boltzmann's constant, familiar from the ideal gas law $PV = NkT$. The result of Rayleigh's derivation gave

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}, \qquad \text{(valid only if } \lambda \text{ is long)} \tag{3.20}$$

which agrees well with the long-wavelength tail of the blackbody radiation curve. However, a severe problem with Rayleigh's result was recognized immediately; his solution for $B_\lambda(T)$ grows without limit as $\lambda \to 0$. The source of the problem is that according to classical physics, an infinite number of infinitesimally short wavelengths implied that an unlimited amount of blackbody radiation energy was contained in the oven, a theoretical result so absurd it was dubbed the "ultraviolet catastrophe." Equation (3.20) is known today as the **Rayleigh–Jeans law.**[16]

Wien was also working on developing the correct mathematical expression for the blackbody radiation curve. Guided by the Stefan–Boltzmann law (Eq. 3.16) and classical thermal physics, Wien was able to develop an empirical law that described the curve at short wavelengths but failed at longer wavelengths:

$$B_\lambda(T) \simeq a\lambda^{-5} e^{-b/\lambda T}, \qquad \text{(valid only if } \lambda \text{ is short)} \tag{3.21}$$

where a and b were constants chosen to provide the best fit to the experimental data.

Planck's Function for the Blackbody Radiation Curve

By late 1900 the German physicist Max Planck (1858–1947) had discovered that a modification of Wien's expression could be made to fit the blackbody spectra shown in Fig. 3.8 while simultaneously replicating the long-wavelength success of the Rayleigh–Jeans law and avoiding the ultraviolet catastrophe:

$$B_\lambda(T) = \frac{a/\lambda^5}{e^{b/\lambda T} - 1},$$

In order to determine the constants a and b while circumventing the ultraviolet catastrophe, Planck employed a clever mathematical trick. He assumed that a standing electromagnetic wave of wavelength λ and frequency $\nu = c/\lambda$ could not acquire just any arbitrary amount of energy. Instead, the wave could have only specific allowed energy values that

[15] This is analogous to standing waves on a string of length L that is held fixed at both ends. The permitted wavelengths are the same as those of the standing electromagnetic waves.

[16] James Jeans (1877–1946), a British astronomer, found a numerical error in Rayleigh's original work; the corrected result now bears the names of both men.

were integral multiples of a minimum wave energy.[17] This minimum energy, a **quantum** of energy, is given by $h\nu$ or hc/λ, where h is a constant. Thus the energy of an electromagnetic wave is $nh\nu$ or nhc/λ, where n (an integer) is the number of quanta in the wave. Given this assumption of quantized wave energy with a minimum energy proportional to the frequency of the wave, the entire oven could not contain enough energy to supply even one quantum of energy for the short-wavelength, high-frequency waves. Thus the ultraviolet catastrophe would be avoided. Planck hoped that at the end of his derivation, the constant h could be set to zero; certainly, an artificial constant should not remain in his final result for $B_\lambda(T)$.

Planck's stratagem worked! His formula, now known as the **Planck function**, agreed wonderfully with experiment, but *only* if the constant h remained in the equation:[18]

$$\boxed{B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}.} \tag{3.22}$$

Planck's constant has the value $h = 6.62606876 \times 10^{-34}$ J s.

The Planck Function and Astrophysics

Finally armed with the correct expression for the blackbody spectrum, we can apply Planck's function to astrophysical systems. In spherical coordinates, the amount of radiant energy per unit time having wavelengths between λ and $\lambda + d\lambda$ emitted by a blackbody of temperature T and surface area dA into a solid angle $d\Omega \equiv \sin\theta \, d\theta \, d\phi$ is given by

$$B_\lambda(T)\, d\lambda \, dA \, \cos\theta \, d\Omega = B_\lambda(T)\, d\lambda \, dA \, \cos\theta \, \sin\theta \, d\theta \, d\phi; \tag{3.23}$$

see Fig. 3.9.[19] The units of B_λ are therefore W m^{-3} sr^{-1}. Unfortunately, these units can be misleading. You should note that "W m^{-3}" indicates power (energy per unit time) per unit area per unit wavelength interval, W m^{-2} m^{-1}, *not* energy per unit time per unit volume. To help avoid confusion, the units of the wavelength interval $d\lambda$ are sometimes expressed in nanometers rather than meters, so the units of the Planck function become W m^{-2} nm^{-1} sr^{-1}, as in Fig. 3.8.[20]

At times it is more convenient to deal with frequency intervals $d\nu$ than with wavelength intervals $d\lambda$. In this case the Planck function has the form

$$\boxed{B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}.} \tag{3.24}$$

[17]Actually, Planck restricted the possible energies of hypothetical electromagnetic oscillators in the oven walls that emit the electromagnetic radiation.

[18]It is left for you to show that the Planck function reduces to the Rayleigh–Jeans law at long wavelengths (Problem 3.10) and to Wien's expression at short wavelengths (Problem 3.11).

[19]Note that $dA \cos\theta$ is the area dA projected onto a plane perpendicular to the direction in which the radiation is traveling. The concept of a solid angle will be fully described in Section 6.1.

[20]The value of the Planck function thus depends on the units of the wavelength interval. The conversion of $d\lambda$ from meters to nanometers means that the values of B_λ obtained by evaluating Eq. (3.22) must be divided by 10^9.