Topics: Robertson-Walker metric, proper distance and redshift and the scale factor, full Friedmann equation

Reading:

- Read sections 4 and 5 of Ch. 23 (pp. 543-49).

Summary of work to submit:

- Nothing to hand in for Tuesday's class.

Overview:

Curvature of spacetime on both small scale and the largest scales is described by a *metric* of spacetime, and on large scales, the Robertson-Walker metric encapsulates both curvature and expansion. We can use the metric to calculate the *proper distance* to a galaxy (or to the CMB's last scattering surface or to the cosmic horizon, for that matter) according to equation 23.59. But to do that, we need to know the history of the scale factor. To compute that, we revisit the Friedmann equation, and see that the energy constant, k, is associated with the geometry – the curvature – of the universe. And that we can't necessarily ignore the cosmological constant, *Lambda*, Λ. To map out the history of the scale factor, then, we need to know how strong Λ is, how much gravity from baryonic and dark matter there is, and also what the photon (and maybe neutrino) energy densities are. And how these values and their associated dynamics, change with time. To do that, we scale everything to the critical density (equation 23.76).

Commentary on the reading, viewing, and other preparation:

The metric of spacetime...think back to Physics 5. The null geodesics that describe photon paths are essentially the 45-degree lines in a spacetime diagram.

The proper distance is a key quantity that we can calculate but can't directly measure. It is the *current* distance to, say, a galaxy whose light we're observing now, which is bigger than the distance of the galaxy at the time the light was emitted.

We can observe a galaxy's redshift, which tells us the scale factor at the time the light was emitted, but to relate that to the proper distance, we need to know the form of $a(t)$.

The power-law example at the end of sec. 4 is just a contrived example, but it shows you in a concrete way how the history of the expansion (i.e. $a(t)$) affects the relationship between a measured redshift and the proper distance and how that result differs from what you'd expect in a static universe.

Note that equation 23.62 has a mistake in it (switching numerator and denominator).

We can continue to discuss the material in §5 on Thursday, but let's try to begin it on Tuesday. The relativistic Friedmann equation has a derivation...that requires an understanding of the formalism of general relativity. But it maps on quite nicely to the Newtonian version we learned last week.

Curvature is more positive the larger the energy density of any component (including Λ) is (see equation 23.72), but some of those component represent attractive forces that slow the expansion while others (Λ!) represent repulsive forces that accelerate the expansion. The chapter closes by noting that we will want to know how each component changes as a function of time (or as a function of the scale factor).

Finally, the Ω formalism expressed in equation 23.76 is widely used and good to get used to. If the universe is flat then $\Omega = 1$.