Topics: orbits and Kepler's laws, mass determinations, eccentric orbits

Reading:

- Ryden and Peterson, Ch. 2, section on Kepler's Laws (pp. 50 - 52; plus Fig. 2.19 on p. 53).
- Ryden and Peterson, Ch. 3, read just the one paragraph at the beginning of §3.1 (on p. 62) about Deriving Kepler's Laws (review the previous two pages, as necessary). Then skim the rest of §3.1; pay special attention to Fig. 3.6 and compare it to Fig. 2.17.
- http://www.nowykurier.com/toys/gravity/gravity.html – Play around with the *gravity simulator* linked from the right side of our class website and see if you can set up a small object in a circular orbit around a big object.

Summary of work to submit:

- Bring your solutions to problem 1 to Tuesday's class.

Overview:

We will start with the $F_{cent} = F_g$ giving $v^2 = GM/r$ for a circular orbit of a lighter object (mass, m) around a heavier object (mass, M) that we ended with last time, and put it into a form that's often more useful – Kepler's third law. We'll discuss the most important and useful aspects of Kepler's laws (relaxing the assumption of $m << M$ and of a circular orbit, and look at a few applications). And we'll also see how the full version (eqn. 3.53 on p. 73) can be used, including cases where orbits aren't circular and also where the orbiting body has a mass comparable to the more massive body.

Commentary on the reading, viewing, and other preparation:

Note in the Ch. 2 reading that Kepler (around 1600) discovered a special case of the third law, purely empirically (not using any physical theory, just trying more-or-less random functions to fit his data on the orbits of the planets). Newton, later in the century (Ch. 3), showed how Kepler's laws flow naturally from an inverse-square law of gravity (we did this in the last class). We will *not* study the derivations of the laws, but rather I will show you a very simple derivation for the special case of a circular orbit of a low-mass object (like the Moon) orbiting a much more massive object, like the Earth. However, do skim the derivations in Ch. 3 to get a flavor of how they're done (e.g. the second law is all about conservation of angular momentum) – but don't spend a lot of time on these derivations; and don't be daunted by the vector calculus either – if you don't fully understand it now, you will by next year.

When you read p. 73 where you'll see that in Newton's version of Kepler's third law, the masses of both objects appear (for planets orbiting the Sun, the masses of the different planets have little effect, and the Sun's mass doesn't have to explicitly appear since it's the same for every planet considered). That's the form you get from the full derivation that you skimmed. Note that this full version comes from the fact that in reality, when two bodies are orbiting each other, they are both orbiting their common center of mass.

Spend a few minutes thinking/looking at how the value of the *eccentricity* affects the nature of an orbit. It's a remarkable fact that the eccentricity doesn't enter into Kepler's third law. Note carefully how the semi-major axis is defined, and where the Sun is (not at the center of planets' elliptical orbits).

Problem 1

(a) Using Newton's law of gravity, show that an apple falling from a tree accelerates at 9.8 m s^{-2}.

(b) Given the Moon's greater distance (how many Earth radii away is it?) compute the acceleration due to the Earth's gravity on the Moon.

(c) Given the Moon's orbital period (27.3 days) and distance, compute its centripetal acceleration.

These last two calculations give the same answer – that's what made Newton realize that his law of gravity was really *universal* when he saw the apple falling from the tree (and perhaps the Moon in the daytime sky).

Finally – the gravity simulator! It is fun to play with. Here's what I'd like you to do: (1) put two relatively large masses on the screen, and watch them accelerate toward each other; (2) put a big mass somewhere on the screen, then choose a smaller mass object and place it a few inches from the first object, giving it some more or less random initial velocity (drag the mass when you place it down) and see what happens; (3) try again but give the small mass an initial velocity that you think will put it into a *circular orbit*; (4) try again until you get something pretty close to circular. It's not easy. But going through this process should really help solidify your understanding of the fact that orbital motion is a *combination of tangential inertia plus the centrally directed force of gravity*. Also note that when you fail to make a circular orbit, you end up with an elliptical orbit (or a hyperbolic orbit). When you do this, you can inspect the orbital motion and see (qualitatively) that Kepler's second law is obeyed (equal areas in equal times, so slow far from the central massive object and fast near it).