

## line profile notes

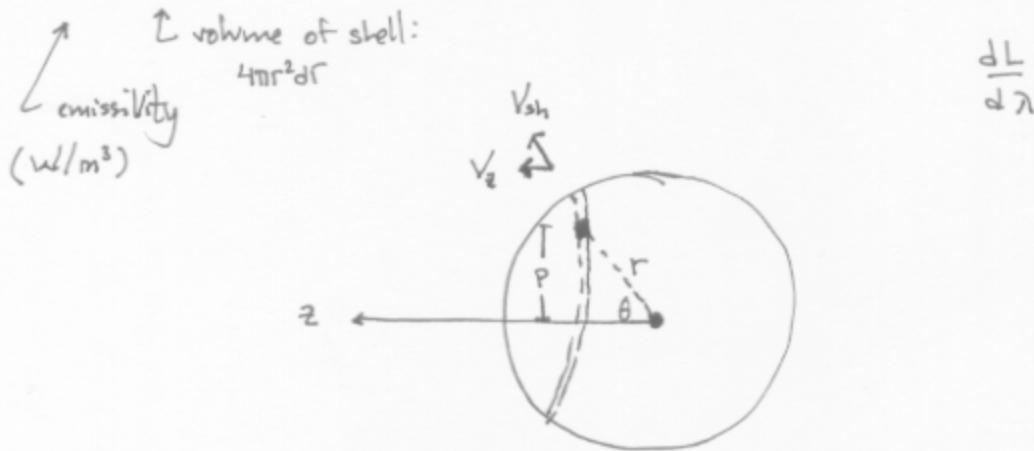
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consider a geometrically thin, uniformly expanding, spherical shell. Its total luminosity,  $L$ , in Watts (Joules/sec), is given by

$$L = \eta \cdot \text{Vol}_{\text{shell}}$$

We want to know the line profile:



- figure out where on the sphere photons with wavelengths in the range  $\lambda + \Delta\lambda$  are emitted
- figure out what fraction of the total volume that sub-volume represents
- (eventually) integrate over spherical shells to generate a total profile  
[we will ignore absorption for the time being]

$$\frac{dL}{d\lambda} = \frac{dL}{d\theta} \frac{d\theta}{d\lambda}$$

from insight that  $\lambda + \Delta\lambda \longleftrightarrow \theta + \Delta\theta$

$\lambda$  considerations:  $\frac{\Delta\lambda}{\lambda_0} = \frac{V_2}{c}$   $\therefore V_2 = \cos\theta V_{\text{sh}}$  later we'll want to scale

$$V_2 = \frac{\Delta\lambda}{\lambda_0} c = \left(1 - \frac{\lambda}{\lambda_0}\right) c$$

(later: ) whoops!  
this should be  $\frac{\lambda}{\lambda_0} - 1$   
so a sign error is introduced!

$$X = \left(1 - \frac{\lambda}{\lambda_0}\right) \frac{c}{V_{\text{sh}}}$$

(see note to the right)

$$X = \frac{V_2}{V_{\text{sh}}} \quad \text{note: } X \text{ goes from } -1 \text{ to } +1$$

$\therefore V_{\text{sh}}$  is the terminal velocity of the wind

so, maybe what we want is  $\frac{dL}{dx} = \frac{dL}{d\theta} \frac{d\theta}{dx}$

[that way our line profile will always be on the domain  $[-1, 1]$ ]

$$V_z = V_{sh} \cos \theta$$

$$X = V_z / V_{\infty}$$

so  $X = \frac{V_{sh}}{V_{\infty}} \cos \theta$  ← we can evaluate  $\frac{d\theta}{dx}$  from this

$$\therefore X = \left(1 - \frac{\lambda}{\lambda_0}\right) \frac{C}{V_{\infty}}$$

now, on to the volume of the sphere — or really, the luminosity of the entire sphere, at a function of  $\theta$

⇒ let's express the area of the sphere by considering lengths of the circular strips of constant  $\theta$  inscribed on the sphere:  $l = 2\pi r$

so area:  $\int_0^{\pi} l r d\theta = \int_0^{\pi} 2\pi r^2 r d\theta$  ; note:  $r = r \sin \theta$   
to thickness  
of each circular  
strip in the  $\theta$ -direction  
 so area =  $\int_0^{\pi} 2\pi r^2 \sin \theta d\theta$

∴ thus volume:  $\int_0^{\pi} 2\pi r^2 \sin \theta d\theta dr$

$$Vol_{shell} = 2\pi r^2 dr \int_0^{\pi} \sin \theta d\theta$$

[but note: we don't integrate over  $dr$  until we add up the contributions from all the shells; here, we're still looking at a single shell]

∴ recalling  $L = \eta \cdot Vol_{shell}$

$$L = 2\pi r^2 dr \eta \int_0^{\pi} \sin \theta d\theta$$

so  $\frac{dL}{d\theta} = 2\pi r^2 dr \eta \sin \theta$

∴  $\frac{dx}{d\theta} = -\frac{V_{sh}}{V_{\infty}} \sin \theta$

[sign error: see note at bottom of p.1 — drop the minus sign at the top of next page]

Thus

$$\frac{dL}{dx} = \frac{dL}{d\theta} \frac{d\theta}{dx}$$

$$= \frac{+2\pi r^2 dr \gamma V_{sh} \sin\theta}{V_{sh} \sin\theta}$$

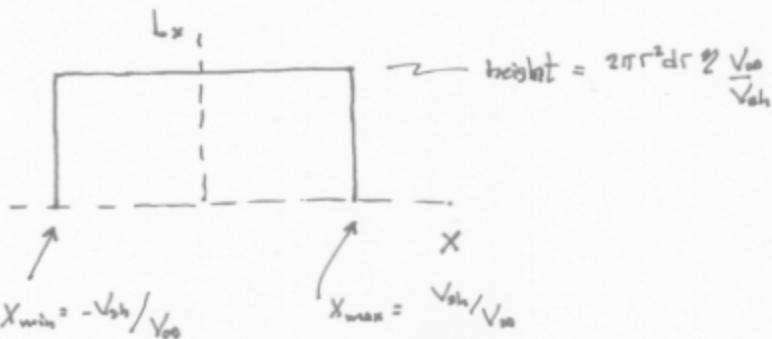
$$\frac{dL}{dx} = 2\pi r^2 dr \gamma \frac{V_0}{V_{sh}}$$

because ours  
is  $\text{W/m}^3$   
this is  $\text{W/m}^3/\text{sterad}$

note:  $\theta$ -dependence (i.e. hence  $\lambda$ -i.e.  $x$ -dependence) drops out

Note: the emissivity we use here differ by a factor of  $4\pi$  from the one introduced in eqn. 2 i.e. defined in eqn. (a) of Zweibel i.e. column 2081

$\therefore$  The line profile is flat-topped



Now, we can model the entire wind by summing up the contributions from many shells:

$$L_x(x) = \sum_{\text{shells}} 2\pi r^2 dr \gamma \frac{V_0}{V_{sh}} \quad \text{if } \frac{V_{sh}}{V_0} < x < \frac{V_{sh}}{V_0}$$

$$= \emptyset \quad \text{otherwise}$$

note that not only are  $r$ :  $V_{sh}$  different for each shell, but so is  $\gamma$ , as follows:

X-ray emission is due to collisions between electrons & ions; doubling the density - for example - doubles the number of electrons AND ions in any given volume i.e. so would quadruple the number (or rate) of collisions. So,  $\gamma = C\rho^2$

(where the constant  $C$  contains all the physics of how collisions generate X-rays)

$$\rho = \frac{M}{4\pi r^2 V}$$

here  $\dot{M}$  - the wind mass loss rate - is a key parameter describing the stellar wind.

Its units are  $M_\odot/\text{yr}$  or  $\text{kg/s}$

If this equation doesn't seem obvious to you - you can check the units : the limits of high  $\dot{M}, r, V$  to see if the corresponding  $\rho$ -limits make sense [ $\rho$  is  $\text{kg/m}^3$  of course, by the way] - consider that the wind mass loss is spread over the surface area of the star, so  $\frac{\dot{M}}{4\pi R_*^2}$  is  $\text{kg/m}^2/\text{s}$  → it's a mass flux

to go from a mass flux crossing some boundary to a density filling an adjacent volume, you have to figure out how far the material crossing the boundary goes in 1 second, i.e. That's just the velocity,  $V$  ( $d = V \cdot t = V \cdot 1 \text{ sec} = V$ ). So the volume filled is  $4\pi R_*^2 V$  ; thus the density is  $\rho = \dot{M} / 4\pi R_*^2 V$ . Now, just replace the star's surface with any (imaginary) sphere above the surface ; you have :

$$\rho = \frac{\dot{M}}{4\pi r^2 V}$$

as long as there are no sinks or sources of mass between the star's surface & the sphere,  $r$ .

[note: so far  $V=\text{constant}$  - as for photont - we have the inverse square law (of light);

i.e. also, you may note a certain similarity to Gauss' law of electrostatics]

OK, so we've got  $\dot{M} = C \rho^2 = C \left( \frac{\dot{m}}{4\pi r^2 V_{sh}} \right)^2$

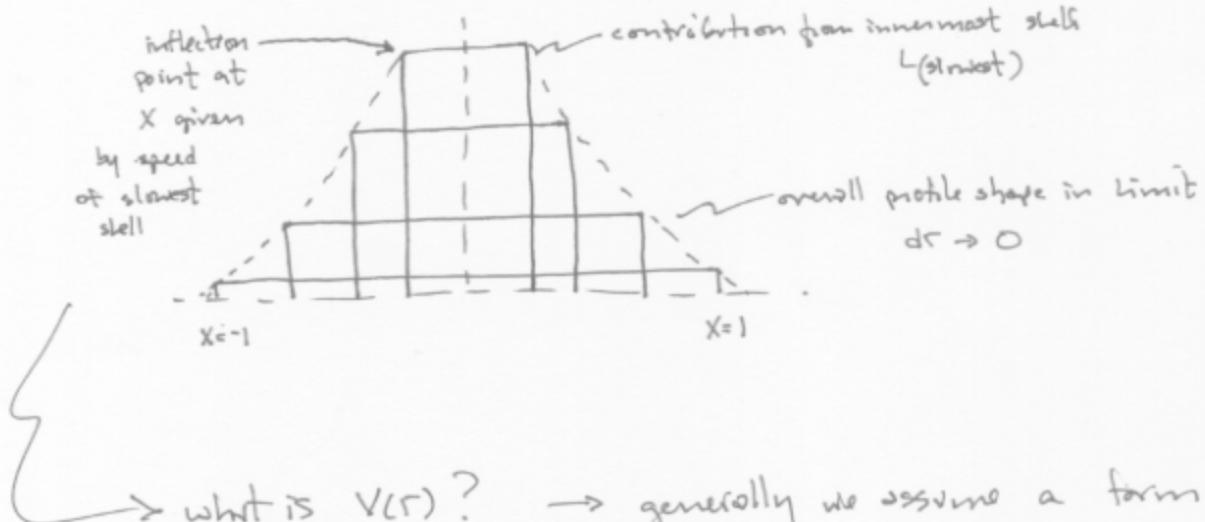
or  $\frac{dL}{dx} = 2\pi r^2 dr \frac{V_{sh}}{V_\infty} C \left( \frac{\dot{m}}{4\pi r^2 V_{sh}} \right)^2$  for a single shell

$$\frac{dL}{dx} = \frac{V_{sh}}{V_\infty} \frac{dr}{8\pi^2 r^2} C \frac{\dot{m}^2}{V_{sh}^2}$$

note: I won't group the two factors of  $V_{sh}$

NOTE: as  $r$  increases, both  $r$  &  $\sqrt{V_{sh}}$   
get bigger, so the height of  
each subsequent profile gets lower

AND <sup>since</sup>  $V_{sh}$  increased, the profiles  
get progressively broader



→ what is  $V(r)$ ? → generally we assume a form

$$V(r) = V_\infty (1 - R_\infty/r)^\beta$$

where  $V_\infty$ , the terminal velocity, has already been introduced  
 $\beta$  is a parameter that varies from wind to wind  
but is generally around  $\beta=0.8$

OK, the expression w/ the summation near the bottom of page 3 should be sufficient — along w/ the expressions for  $\eta, p, \dot{v}, V$  as functions of  $r$  — to calculate any line profile ... if we ignore absorption.

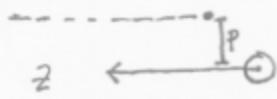
Note: we might want to include occultation — the star itself blocking the back of the wind (or a cylinder of it) — we could think about this in the context of the previous analysis as truncating the  $\int d\theta$  integral of each shell at  $\theta_{\max}$  where:

$$\sin(\pi - \theta_{\max}) = \frac{R_\star}{r_{\text{shell}}}$$

Now, it turns out that if you want to include absorption things become rather messy. The basic reason for this is that the emission — as we've seen — is spherically symmetric, while the absorption is cylindrically symmetric.

The key quantity we need to model the absorption is the optical depth,  $\tau$ , between any arbitrary point in the wind and our telescope:

[def'n of  $\tau \equiv \int p K dx$  — so, need  $p(r)$  to evaluate, but really  $p(z, p)$ ]  
 $\uparrow$  opacity,  $K$  ( $\text{m}^2/\text{kg}$ )



The cylindrical coord system

uses  $z, p, \theta$

$\uparrow$   $\tau$  not the same  $\theta$  as  
often called  $R$   
in spherical  
polar  
coordinates;  
in fact, more  
like  $\phi$

we say, the optical depth integral is

"along a ray of constant impact parameter,  $p$ "

OK, we'll leave absorption for later

but we'll close by trying to draw closer parallels between  
 the approach we use here & the formalism used in Owocki & Cohen 2001.  
 That formalism is especially good for incorporating absorption  
 but it's also good for a completely optically thin ( $\tau=0$ ;  
 no absorption) model, too.

→ eqn. 1 on p.1109 of Owocki & Cohen 2001:

$$L_n = 8\pi^2 \int_{-1}^1 d\mu \int_{R_\infty}^\infty dr r^2 \gamma_n(\mu, r) e^{-\tau(\mu, r)}$$

is very close to what we have — see the 3rd-from-last  
 equation on p.2 of these notes:

$$L = 2\pi r^2 dr \gamma \int_0^\pi \sin\theta d\theta$$

this is for a single shell — i.e., before we did the differentiation to  
 get  $\frac{d\gamma}{d\theta}$  → this is where we need to start, because  
 to include absorption we need to wait to integrate  $\gamma$  over  $\theta$   
 until we have an explicit  $\theta$ -dependent  $\tau$  specified.

Now,  $\mu \equiv \cos\theta$  so  $d\mu = -\sin\theta d\theta$

$$\text{thus, } \int_0^\pi \sin\theta d\theta = \int_{-1}^1 d\mu$$

plus, I already noted the extra factor of  $4\pi$  in the def'n of  $\gamma$   
 so, our expression is  $L = 2\pi r^2 dr \gamma \int_0^\pi \sin\theta d\theta$   
 $L = 8\pi^2 \int_{-1}^1 d\mu r^2 \gamma d\mu$  w/ the new normalization

this is from one shell; integrating over shells from  $r=R_\infty$  to  $\infty$ :

$$L = 8\pi^2 \int_{-1}^1 d\mu \int_{R_\infty}^\infty dr r^2 \gamma$$

↑ this is eqn. 1 in the paper,  
 except for the absorption  
 factor  $e^{-\tau}$ .