

# Quantifying mass-loss rate and wind porosity using the X-ray emission line profiles of $\zeta$ Puppis

David H. Cohen,<sup>1\*</sup> Maurice A. Leutenegger,<sup>2</sup> Emma E. Wollman,<sup>1</sup>  
Stanley P. Owocki,<sup>3</sup> Richard H. D. Townsend<sup>3,4</sup>

<sup>1</sup>*Swarthmore College, Department of Physics and Astronomy, Swarthmore, Pennsylvania 19081, USA*

<sup>2</sup>*NASA/Goddard Space Flight Center, Laboratory for High Energy Astrophysics, Code 622, Greenbelt, Maryland 20771, USA*

<sup>3</sup>*University of Delaware, Barol Research Institute, Newark, Delaware 19716, USA*

<sup>4</sup>*University of Wisconsin, Department of Astronomy, Madison, 475 N. Charter St., Madison, Wisconsin 53706, USA*

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## ABSTRACT

We fit X-ray line profile models, including the effects of large-scale wind porosity, to the high-resolution *Chandra* spectrum of the O4 supergiant  $\zeta$  Pup. We find that models that include porosity provide a somewhat worse fit to the data than models where the X-ray opacity is purely atomic and which do not include porosity. We also fit a porous model with oblate clumps, and find that it provides much worse fits to the data. From the fits to 14 emission lines between 6 and 22 Å we find a modest wavelength dependence in the optical depth, which is consistent with the expected atomic opacity and inconsistent with a porosity dominated medium, where the geometrical cross section of the clumps governs the effective opacity. From the fits to these lines, we derive a mass-loss rate of  $3.0 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ , which represents a factor of  $\sim 3$  reduction of the traditional mass-loss rate derived assuming no wind clumping, and is consistent with more recent determinations that include small-scale clumping.

**Key words:** stars: early-type – stars: mass-loss – stars: winds, outflows – stars: individual:  $\zeta$  Pup – X-rays: stars

## 1 INTRODUCTION

The dense and highly supersonic radiation-driven winds of O stars are generally supposed to be the site of X-ray production in these massive stars. Broadened X-ray emission line profiles ( $v_{\text{hwhm}} \approx 1000 \text{ km s}^{-1}$ ), first measured with *XMM-Newton* and *Chandra* early in this decade, provide direct evidence for hot plasma kinematics consistent with the same beta velocity law that describes the bulk of the cool ( $T < T_{\text{eff}}$ ) wind (Kramer et al. 2003). This hot, X-ray emitting plasma is thought to be produced by shock heating of a small fraction of the wind to temperatures of a few million K, and it is generally supposed that the line-driven instability (LDI) is the cause of the shocks (Owocki et al. 1988; Feldmeier et al. 1997; Dessart & Owocki 2003). The high-resolution X-ray spectra not only provide information about the hot, X-ray emitting wind component, they also provide important information about the bulk, cool wind component which attenuates the emitted X-rays.

The early O supergiants, with the highest mass-loss

rates, are expected to have winds that are quite optically thick to X-rays (Hillier et al. 1993). One readily observable effect of optically thick winds is the apparent blue shift and asymmetry of emission lines, which arises because red shifted X-rays emitted from the back of the wind are preferentially absorbed compared to blue shifted photons from the front hemisphere of the wind. The degree of blue shift and asymmetry is proportional to a single parameter that describes the optical depth along the central ray,  $\tau_* \equiv \frac{\dot{M}\kappa}{4\pi R_* v_{\infty}}$ . So, the line profile shape, through  $\tau_*$ , provides a powerful diagnostic of the mass-loss rate.

This X-ray line profile mass-loss rate diagnostic is independent of the small-scale clumping (microclumping) that affects density-squared diagnostics of mass loss, like H $\alpha$  and radio free-free emission. Evidence has recently emerged that microclumping does affect these traditional mass-loss rate diagnostics, and the mass-loss rates of OB stars are starting to be revised downward. There is a need for independent confirmation of lowered mass-loss rates and X-ray line profiles can potentially provide that. The initial quantitative analysis of resolved X-ray profiles, which are less blue

\* E-mail: cohen@astro.swarthmore.edu

shifted and asymmetric than expected, indicated that lower mass-loss rates are favored. One goal of this paper is, for the first time, to make a quantitative determination of an O star’s mass-loss rate from the analysis of its X-ray line profiles.

It has been shown, though, that large-scale clumping – so-called macroclumping – can reduce the effective opacity of the wind and could, in principle, account for the only modestly shifted and asymmetric X-ray profiles that are observed in many O stars (Feldmeier et al. 2003; Oskinova et al. 2004, 2006; Owocki & Cohen 2006). This effect only occurs once individual clumps become optically thick in the X-ray continuum, so that opacity can be effectively hidden in the interior of clumps. When this criterion is met, we say that the wind is porous. Photon escape from the wind is enhanced by this porosity and the wind’s effective opacity is reduced. The key parameter that describes the effects of porosity on X-ray line profiles is the porosity length,  $h \equiv \ell/f = L^3/\ell^2$ , where  $\ell$  is the characteristic clump size scale,  $L$  is the characteristic interclump spacing, and  $f$  is the volume filling factor of clumps (the interclump medium is assumed to contain negligible mass). In the limit of completely optically thick, geometrically thin clumps (“shell fragments” in the parlance of Feldmeier et al. (2003)), the porosity length is also the photon mean free path in the radial direction, or the radial interclump spacing.

Another goal of this paper, then, is to quantitatively assess the evidence for wind porosity in the observed X-ray line profiles. We will do this in two different ways: (1) we will fit models of line profiles that include both the mass-loss rate (via  $\tau_*$ ) and the porosity (via  $h_\infty$ , the porosity length in the far wind) and jointly constrain the values of these model parameters; and (2) we will fit line-profile models without porosity to the many lines in the spectrum and assess the evidence for a trend in  $\tau_*$  with wavelength. In the case of porosity-dominated effective wind opacity (optically thick macroclumping), the cross section to X-rays is governed by the geometric cross section of the clumps, which is wavelength independent, and therefore no trend is expected. In the case where porosity is negligible, the wind opacity is controlled by atomic cross sections (due to photoelectric absorption) of abundant heavy elements in the wind, which is wavelength dependent. We note, though, that the wavelength dependence of the wind opacity is complicated and dependent on the abundances in the wind and on the ionization balance, and therefore we include detailed modeling of the wind ionization and opacity in this paper.

Even in the case that the effects of porosity and atomic opacity cannot be definitively disentangled, we can still quantitatively explore the trade-offs between these two effects. Specifically, in this paper, we assess for the first time just how much porosity (what values of  $h_\infty$ ) are required to fit the data with the traditional, higher mass-loss rates. After making these determinations for the strong lines in the *Chandra* spectrum of  $\zeta$  Pup, we assess the realism of the derived porosity lengths in light of the small-scale wind structure generated in state-of-the-art two-dimensional radiation hydrodynamics simulations of the LDI in O star winds (Dessart & Owocki 2003).

Finally, it has been suggested that oblate (flattened) clumps, with their surfaces oriented parallel to the star’s surface, are especially good at producing symmetric and un-

shifted line profiles (Oskinova et al. 2006). We therefore include this effect in our line-profile model and fit these models to the data to, for the first time, quantitatively assess whether oblate clumps can in fact account for the observed profile shapes.

The paper is organized as follows: We begin by describing the *Chandra* data set and defining a sample of well behaved emission lines for our analysis in §2. We also briefly evaluate the stellar and wind properties of  $\zeta$  Pup in this section. In §3 we briefly describe the empirical line profile model for X-ray emission lines, with and without porosity. We also introduce our oblate clump porosity line profile model in this section. And we show how porosity affects X-ray emission line profiles, including how its effects on the profile morphologies differ from those caused by reduced mass-loss rates. In §4 we describe our procedure for analyzing data with the line-profile models presented in the previous section. In §5 we present our results, and in §6 we discuss their implications, including a consideration of wavelength-dependent wind opacity and the results of simulations of wind structure induced by the line-driven instability. In §7 we conclude that the line profile shapes in the *Chandra* grating spectrum of  $\zeta$  Pup require a mass-loss rate of  $3.0 \times 10^{-6} M_\odot \text{ yr}^{-1}$ , and that higher mass-loss rates can be accommodated if the effective optical depth of the wind is reduced by porosity, but that unrealistically large values of the porosity length are required for consistency with the literature mass-loss rates. We also conclude that the wavelength dependence of the profile properties is consistent with mass-loss rate reduction and not with the gray effective opacity implied by significant porosity effects. And, finally, we conclude that line-profile models with flattened clumps do not provide good fits to the data.

## 2 THE *Chandra* GRATING SPECTRUM OF $\zeta$ PUP

### 2.1 The data

All the data we use in this paper was taken on 28-29 March 2000 in a single, 68 ks observation using the *Chandra* High-Energy Transmission Grating Spectrometer (HETGS) in conjunction with the Advanced CCD Imagine Spectrometer (ACIS) detector in spectroscopy mode. This is a photon counting instrument with an extremely low background and high spatial resolution ( $\approx 1''$ ). The first-order grating spectra we analyze have a total of 21,684 counts, the vast majority of which are in emission lines, as can be seen in Fig. 1. We modeled every line in the two spectra, as we describe in §4 and §5, and indicate in this figure which of the lines we deemed to be reliable. We only include lines in our analysis that are not so weak or severely blended that interesting parameters of the line-profile models cannot be reliably constrained.

The HETGS assembly has two grating arrays - the Medium Energy Grating (MEG) and the High Energy Grating (HEG) - with spectral resolutions of  $0.0023 \text{ \AA}$  and  $0.0012 \text{ \AA}$ , respectively. This corresponds to a resolving power of  $\mathcal{R} \approx 1000$ , or a velocity of  $300 \text{ km s}^{-1}$ , at the longer wavelength end of each grating. The wind-broadened X-ray lines of  $\zeta$  Pup are observed to have  $v_{\text{fwhm}} \approx 2000 \text{ km s}^{-1}$ , and so

are very well resolved by *Chandra*. The wavelength calibration of the HETGS is accurate to  $50 \text{ km s}^{-1}$ .

The two gratings, detector, and telescope assembly have significant response from roughly  $2 \text{ \AA}$  to  $30 \text{ \AA}$ , with typical effective areas of tens of  $\text{cm}^2$ , and a strong function of wavelength. In practice, the shortest wavelength line with significant flux in the relatively soft X-ray spectra of O stars like  $\zeta$  Pup is the Si XIV Lyman-alpha line at  $6.182 \text{ \AA}$ , and the longest wavelength line is the N VII Lyman-alpha line at  $24.781 \text{ \AA}$ . The HEG response is negligible for lines with wavelengths longer than about  $16 \text{ \AA}$ .

The X-ray spectrum of  $\zeta$  Pup consists primarily of emission lines from H-like and He-like ionization stages of N, O, Ne, Mg, and Si, and numerous L-shell lines of iron, primarily Fe XVII. The Ly $\alpha$  lines and often the  $\beta$  and even  $\gamma$  lines of the Lyman series are seen for the H-like ions. There is a weak bremsstrahlung continuum beneath these lines. Overall, the spectrum is consistent with an optically thin, thermal plasma in ionization equilibrium with a range of temperatures from one to several million degrees present. It is possible that there are deviations from equilibrium, although the spectrum is not of high enough quality to show this. There is some evidence from the *XMM-Newton* RGS spectrum that a few of the emission lines are optically thick (Leutenegger et al. 2007); a possibility we will take into account when modeling those lines.

## 2.2 The star

$\zeta$  Puppis is nearby ( $d = 335_{-11}^{+12} \text{ pc}$ )<sup>1</sup>, single, runaway early O supergiant that shows the enhanced nitrogen and deficient carbon and oxygen that is indicative of CNO cycle processed material. Helium is also overabundant. The star's rapid rotation may explain the photospheric abundance anomalies, though it has also been claimed that it had a close binary companion that exploded as a supernova, rendering it a runaway and perhaps explaining its anomalous abundances. Detailed spectral synthesis has been carried out from the UV to the IR to determine the stellar and wind properties of  $\zeta$  Pup, which we list in Table 1. Most of these are taken from Puls et al. (2006). There is a range of wind property determinations in the extensive literature on  $\zeta$  Pup. The terminal velocity of the wind may be as low as  $2100 \text{ km s}^{-1}$ , and as high as  $2485 \text{ km s}^{-1}$ . Mass-loss rate determinations vary as well, partly because of the uncertainty in the distance, but also because each mass-loss rate diagnostic is subject to uncertainty: density squared diagnostics like H $\alpha$  and free-free emission are affected by clumping, no matter the size scale and optical depth of the clumps. Mass-loss rates from UV absorption lines are subject to uncertain ionization corrections. In the last few years, microclumping has started to be taken into account when deriving mass-loss rates from both density-squared diagnostics and UV absorption diagnostics. We list several mass-loss rate determinations in the

**Table 1.** Stellar and wind parameters adopted from Puls et al. (2006)

parameter	value
Sp. Type	O4 If
Mass <sup>a</sup>	$53.9 M_{\odot}$
$T_{\text{eff}}$	39000 K
$R_{*}$	$18.6 R_{\odot}$
$v_{\text{rot}} \sin i$	—
$v_{\infty}$	$2250 \text{ km s}^{-1}$
$\beta$	0.9
$\dot{M}^b$	$8.3 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$
$\dot{M}^c$	$4.2 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$
$\dot{M}^d$	$1.5 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$

<sup>a</sup> From Repolust et al. (2004).

<sup>b</sup> Unclumped value from Puls et al. (2006).

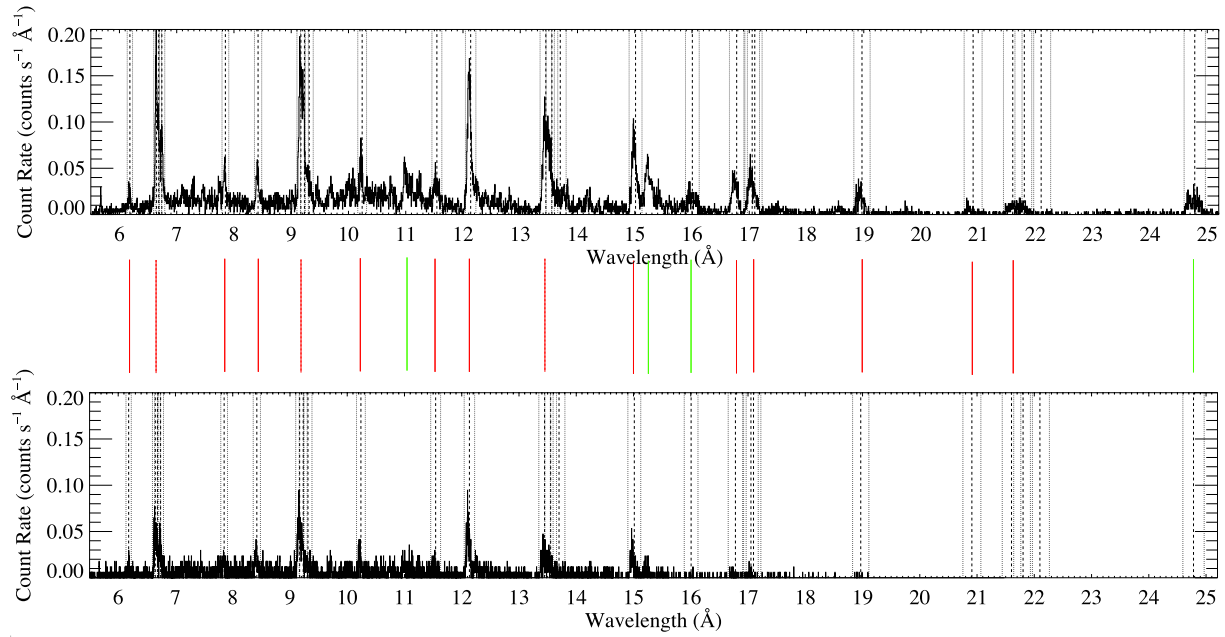
<sup>c</sup> Also from Puls et al. (2006), but the minimum clumping model, in which the far wind, where the radio emission arises, is unclumped, but the inner wind, where the H $\alpha$  is produced is clumped. Note that the methodology of Puls et al. (2006) only enables a determination to be made of the relative clumping in these two regions of the wind.

<sup>d</sup> From Bouret et al. 2008, from detailed UV spectral modeling, assuming clumping.

table, with notes about the assumptions behind each determination. The X-ray line profile diagnostics of mass-loss rate that we employ in this paper are not directly affected by microclumping; only by macroclumping and the associated porosity.

The star shows periodic variability in various UV wind lines as well as H $\alpha$ . Its broad-band X-ray properties are normal for an O star, with  $L_x \approx 10^{-7} L_{\text{Bol}}$  and a soft spectrum, dominated by optically thin thermal line and free-free emission from plasma with a temperature of a few million degrees. The emission measure filling factor of the wind is small, roughly one part in  $10^3$ . Weak soft X-ray variability, with an amplitude of 6 percent, and a period consistent with the 18 hr H $\alpha$  period, was detected with *ROSAT*. This low-level variability appears not to affect the *Chandra* data.

<sup>1</sup> The original Hipparcos distance determination had rather large error bars; this value is from a recent reanalysis of the data. The derived stellar parameters and mass-loss rate depend on the distance. And the distance also has implications for the origin of  $\zeta$  Pup.



**Figure 1.** The entire usable portions of the MEG (top) and HEG (bottom) first order spectra of  $\zeta$  Pup. The binning is native (2.5 mÅ for the HEG and 5 mÅ for the MEG). Vertical dashed lines in the data panels themselves represent the laboratory rest wavelengths of important lines. The lighter dotted lines on either side represent the Doppler shifts associated with the star's terminal velocity. Bold vertical lines between the two spectral plots indicate the lines we successfully fit with profile models (solid red) and lines we attempted to fit but which were too blended to extract meaningful model parameters (solid green). The helium-like triplets are indicated by dotted lines. For these, and all blended emission lines, we show only one of these red or green lines, and align it with the bluest emission line in the blend.

### 3 THE EMISSION LINE PROFILE MODEL AND POROSITY

The non-porous model we fit to each line first was first described by Owocki & Cohen (2001). It is a simple, spherically symmetric model that assumes that the local emission scales as the ambient density squared and that the many sites of hot, X-ray emitting plasma is statistically distributed throughout the wind above some onset radius,  $R_o$ , which is expected to be at  $R_o \approx 1.5 R_*$ . Attenuation of the emitted X-rays occurs in the bulk, cool ( $T \approx T_{\text{eff}}$ ) wind component via photoelectric absorption, mainly out of the inner shell of elements C through Si. and also out of the L-shell ( $n = 2$ ) of Fe. We assume that the atomic opacity of the cool wind, while a function of wavelength, does not vary significantly with radius. We further assume a beta-velocity law for both wind components, with  $\beta = 1$  and  $v_\infty = 2250 \text{ km s}^{-1}$  as given by UV observations. Note that the assumptions behind the model are quite general, and don't assume any particular physical mechanism for the heating of the plasma, but that the frequency-dependent radiation transport is solved exactly.

We set the problem up in spherical coordinates, with the origin at the center of the star and the observer at  $z = \infty$ . We integrate the specific intensity along rays of given impact parameter, and then integrate over rays. Treating the problem in spherical coordinates and integrating over the volume of the wind, we have:

$$L_\lambda = 8\pi^2 \int_{-1}^{+1} d\mu \int_{R_o}^{\infty} \eta_\lambda(\mu, r) r^2 e^{-\tau(\mu, r)} dr, \quad (1)$$

X-ray line profile. The angular coordinate,  $\mu \equiv \cos \theta$ ,  $\eta_\lambda$  is the wavelength dependent emissivity that accounts for the Doppler shift of the emitting parcel of wind material (which is completely determined, under the assumptions of spherical symmetry and the velocity law) according to its location,  $\mu$  and  $r$ . It also has a radius dependence due to the fact that it is proportional to the square of the ambient plasma density. The optical depth is given by  $\tau$ . Its expression is more physically reasonable in cylindrical coordinates, as follows:

$$\tau(\mu, r) \equiv t(p, z) = \int_z^\infty \kappa \rho(r') dz', \quad (2)$$

where the dummy radial coordinate is given by  $r' \equiv \sqrt{z'^2 + p^2}$ . The opacity,  $\kappa$ , does not vary significantly across a line (it is due to continuum processes, recall). And using the continuity equation and the beta-velocity law of the wind, we have:

$$t(p, z) = \tau_* \int_z^\infty \frac{R_* dz'}{r'^2 (1 - R_*/r')^\beta}. \quad (3)$$

We account for occultation of the back of the wind by the star by setting this optical depth integral to  $\infty$  when  $p < R_*$  and  $z < \sqrt{R_*^2 - p^2}$ . The constant at the front of eq. 3,  $\tau_* \equiv \frac{\dot{M} \kappa}{4\pi R_* v_\infty}$ , is the fiducial optical depth and is equivalent to the optical depth value along the central ray, integrated down to the stellar surface, in the case where  $v = v_\infty$ . This quantity,  $\tau_*$ , is the single parameter that describes the X-ray attenuation and which governs the shifted and asymmetric form of the line profiles.

We note that the optical depth integral, while generally requiring numerical integration, can be done analytically for integer values of  $\beta$ . We use  $\beta = 1$  throughout this paper (though we report on tests we did for non-integer  $\beta$  values in §4.2), and for that value of the parameter, the optical depth integral evaluates as:

$$t(p, z) = \frac{\tau_*}{z_*} \left( \arctan \frac{\mu'}{z_*} + \arctan \frac{z'}{z_*} \right)_{z' \rightarrow z}^{z' \rightarrow \infty}, \quad (4)$$

where  $z_* \equiv \sqrt{p^2 - R_*^2}$ .

The intrinsic line profile function we assume for the emissivity at each location is a delta function that picks out the Doppler shift line resonance,

$$\eta \propto \delta(\lambda - \lambda_o(1 - \mu v(r)/c)). \quad (5)$$

This assumption is justified because the actual intrinsic line broadening is dominated by thermal broadening, with is very small compared to the Doppler shift caused by the highly supersonic wind flow.

Calculating a line profile model, then, amounts to solving equations 1 and 3 for a given set of parameters,  $R_o$ ,  $\tau_*$ , the normalization (which determines the value of  $\eta$ ), and an assumed wind velocity law, described by  $\beta$  and  $v_\infty$ . This last parameter influences the emissivity term through its effect on the Doppler shift as a function of radius and spherical polar angle. And for our choice of  $\beta = 1$ , we use eq. 4 rather than eq. 3.

The model produces broad emission lines, where the overall width, for an assumed wind velocity law, is governed by the parameter  $R_o$ . The closer to the star's surface  $R_o$  is, the more emission from low-velocity wind material, which contributes to the line profile only near the center of the

where  $L_\lambda$  is the luminosity per unity wavelength; it is the

line. The larger  $R_o$  is, therefore, the broader the line profile. The value of  $\tau_*$  affects the line's blue shift and asymmetry. The higher its value, the more blue shifted and asymmetric the profile. The interplay of the two parameters can be seen in figure 2 of Owocki & Cohen (2001).

A simple profile model that includes the effects of large-scale porosity was introduced by Owocki & Cohen (2006), and we adopt that model here and use it for the first time to quantitatively fit data. The key insight of that paper was that a single parameter describes the reduction in the effective opacity, in the presence of porosity. This parameter, the porosity length,  $h \equiv \ell/f$ , where  $f$  is the volume filling factor of clumps<sup>2</sup>, has already been briefly described in the introduction. In the case of significant porosity, where individual clumps are very optically thick, the effective opacity of the clump is simply its macroscopic cross section to its mass,

$$\kappa_{\text{eff}} \equiv \frac{\ell}{m_{\text{clump}}} = \frac{\kappa}{\tau_{\text{clump}}}, \quad (6)$$

where

$$\tau_{\text{clump}} = \kappa \rho_{\text{clump}} \ell = \kappa \langle \rho \rangle h, \quad (7)$$

where  $\kappa$  is the atomic opacity,  $\ell$  is the size scale of the clumps, and  $\langle \rho \rangle$  is the radius-dependent average wind velocity (including the clumps and interclump medium). It comes from the mass-loss rate, terminal velocity, stellar radius, and the continuity equation.

We assume that the porosity length itself is a function of radius, and a reasonable assumption (though not the one used in Owocki & Cohen (2006)) is that the porosity length scales with the local velocity,

$$h = h_\infty (1 - R_*/r)^\beta. \quad (8)$$

Note that this is equivalent to the parameterization of the ‘‘clump release frequency’’ employed by ?, although we retain the porosity length formalism of Owocki & Cohen (2006) as it is more generalizable and also because using a length scale to describe the clumping facilitates comparisons to other determinations of clumping, especially numerical models.

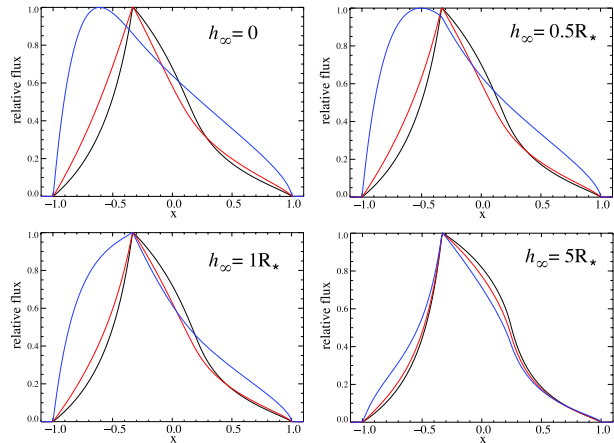
In this case, for spherical clumps, the effective opacity is assumed to take the form,

$$\kappa_{\text{eff}} = \frac{\kappa}{1 + \tau_{\text{clump}}}, \quad (9)$$

and the optical depth integral becomes, :

$$t(p, z) = \tau_* \int_z^\infty \frac{R_* dz'}{(r'^2 + \tau_* h_\infty)(1 - R_*/r')^\beta}. \quad (10)$$

<sup>2</sup> We note that the filling factor, is the reciprocal of the clumping factor, the square root of which represents the mass-loss rate correction factor when clumping is taken into account where it had previously been neglected. Put another way, the factor  $1/\sqrt{f}$  represents the mass-loss rate overestimate if density-squared mass-loss diagnostics are used with clumping neglected.



**Figure 2.** Line profile models that incorporate isotropic porosity. Each panel shows profiles with  $R_o = 1.5 R_*$  and three different values of  $\tau_*$ ,  $\tau_* = 1, 2, 8$ . The terminal porosity length increases from zero in the top left panel (so, these models are non-porous) to  $h_\infty = 5 R_*$ . Note that the effects of porosity are not significant until the porosity length is of order the stellar radius.

## 4 MODEL-FITTING PROCEDURE AND ANALYSIS

### 4.1 Demonstration with one representative emission line

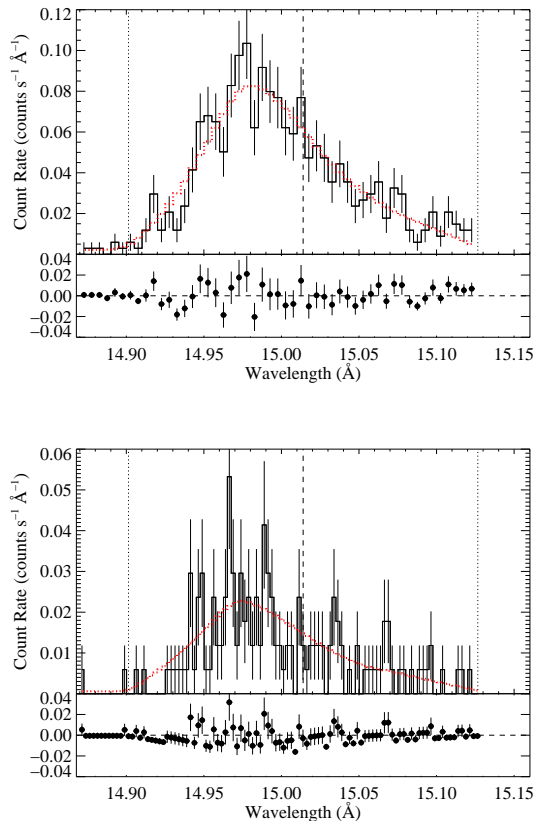
For each line in the spectrum, we first fit the non-porous (Owocki & Cohen 2001) profile model, described by equations 1 and 3, to the data. Note that this model has only three free parameters: the fiducial optical depth,  $\tau_*$ , the minimum radius of X-ray emission,  $R_o$ , and the normalization of the line. After this, we fit the porous model with spherical clumps (Owocki & Cohen 2006), and lastly we fit the porous model with flattened clumps. These two porous models each have only one additional free parameter, the terminal porosity length,  $h_\infty$ , described by equation 8.

We begin the analysis procedure for each line by fitting the weak continuum in two regions, one to the blue side of the line and one on the red side (but excluding the wavelength range of the line itself). We assume the continuum is flat over this restricted wavelength region. We then fit the emission line over a wavelength range that is no broader than the line itself (and sometimes even narrower, due to blends with nearby lines, which induce us to exclude contaminated portions of the line in question). The model we fit to each line is the sum of the empirical line profile model(s) we described in the previous section and the continuum model determined from the fit to the two spectral regions near the line. Note that the inclusion of the continuum does not introduce any new free parameters.

We fit the wind profile plus continuum model to both the MEG and HEG data (positive and negative first orders) simultaneously, if the HEG data are of good enough quality to warrant their inclusion (generally true only for lines shorter than about 16 Å), and to the MEG data only if they are not. We use the C statistic (Cash 1979) as the fit statistic. This is the maximum likelihood statistic for data with Poisson distributed errors, which our photon-counting X-ray spectra are. Note that the maximum likelihood statistic for Gaussian distributed data is the well-known  $\chi^2$  statistic, but it is not valid for these data, which have many bins with only a few counts, especially in the diagnostically powerful wings of the profiles.

We determine the best-fit model by minimization of the C statistic using the *fit* task in XSPEC. Once it is found, the uncertainties on each model parameter are assessed using the  $\Delta\chi^2$  formalism<sup>3</sup> outlined in chapter 15 of Press et al. (2007), which is also valid for  $\Delta C$ . We test each parameter one at a time, stepping through a grid of values and, at each step, refitting data while letting the other model parameters be free to vary. The 68 percent confidence limits determined in this manner are what we report as the formal uncertainties in the tables of fitting results in the next section. We also examine the confidence regions in two-dimensional subspaces of the whole parameter space in order to look for correlations among the interesting parameters.

We will use the relatively strong and unblended Fe XVII line at 15.014 Å to demonstrate this fitting process. We show

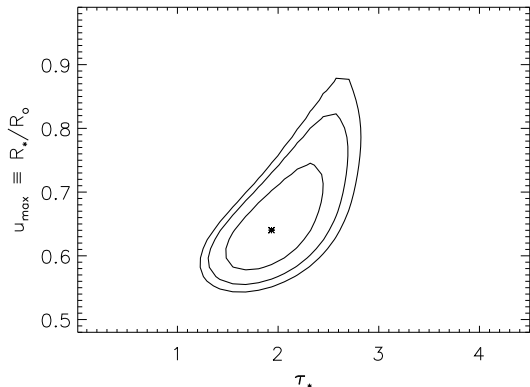


**Figure 3.** The Fe XVII line at 15.014 Å in the MEG (top) and HEG (bottom), with the best-fit non-porous model superimposed. We have not done any rebinning of the data. The error bars represent Poisson, root-N, statistics. The dashed vertical line indicates the laboratory rest wavelength of the emission line, and the two dotted vertical lines indicate the wavelengths associated with the Doppler shift due to the stellar wind terminal velocity of 2250 km s<sup>-1</sup>. The model is shown as a (red) smooth histogram, while the data are shown as a (black) choppy histogram with error bars. The fit residuals are shown in the horizontal windows below the data.

the MEG and HEG data for this line, along with the best-fit model (the set of model parameters,  $\tau_*$ ,  $R_o$ , and normalization which minimizes the C statistic) in Fig. 3. The best-fit model parameters are:  $\tau_* = 1.97$ ,  $R_o = 1.53 R_*$ , and a normalization of  $5.24 \times 10^{-4}$  photons s<sup>-1</sup> cm<sup>-2</sup>. Using the  $\Delta C$  criterion and testing each of these parameters one at a time, we find that the 68 percent confidence limits on the fit parameters are  $1.63 < \tau_* < 2.35$ ,  $1.38 < R_o/R_* < 1.65$ , and  $5.04 \times 10^{-4} < \text{norm} < 5.51 \times 10^{-4}$ . The confidence limits should be thought of as probabilistic statements about the chance that the true parameter values lies within the given range, given the physical assumptions of the model.

In Fig. 4 we show 68, 90, and 95 percent confidence limits in two-dimensional  $\tau_*$ ,  $R_o$  parameter space. We calculate a 35 by 35 grid of models, optimizing the other free parameters (just the normalization, in this case) at each point in the grid, and use values of  $\Delta C = 2.30, 4.61, 6.17$  to define the extent of the confidence limits. Plots such as this one are a good means of examining correlations between model parameters, in terms of their abilities to produce similar

<sup>3</sup> This criterion is a specific numerical value of  $\Delta C \equiv C_i - C_{\min}$  for model realization  $i$ , where  $C_{\min}$  is the C statistic value for the best-fit model.



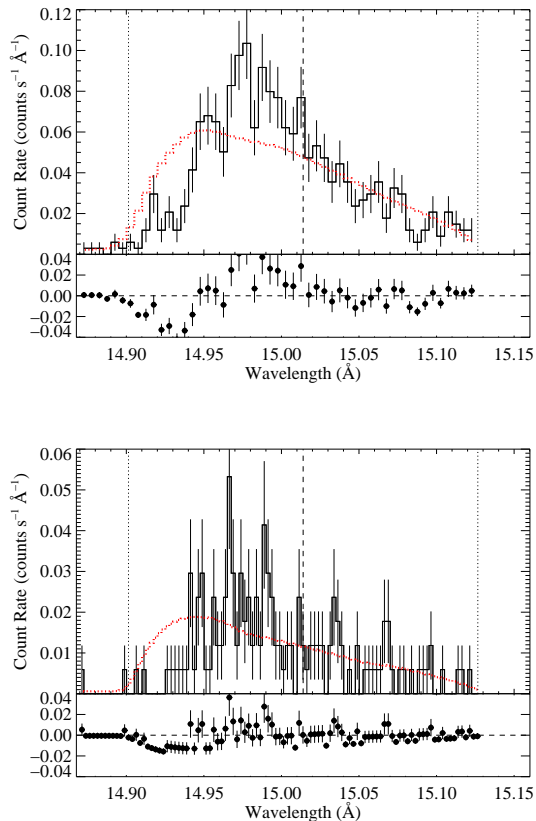
**Figure 4.** Confidence contours (68, 90, and 95 percent) for the non-porous model fitting of the the Fe XVII line at 15.014 Å. The best-fit, shown in Fig. 3, is represented by the asterisk. Note that we plot this, and all other confidence plots that involve the inner radius,  $R_o$ , in terms of  $u_{\max}$ , or  $u_o \equiv R_*/R_o$ .

features in the line profiles. We can see what the tradeoffs are between parameters in a quantitative way. For example, there is a slight correlation between  $u_o$  and  $\tau_*$  evident in the figure. High values of  $u_o$  ( $R_o$  close to  $R_*$ ), reduce emission on the line wing relative to the core (more emitting material at low velocity). So although high values of  $u_o$  (hot plasma as close as 1.2  $R_*$ ) are allowed at the 95 percent confidence limit, they require a large wind optical depth,  $\tau_* \approx 3$ , to compensate. High  $\tau_*$  makes lines more blue-shifted and asymmetric, increasing the emission on the line wing relative to the core.

The value of  $\tau_*$  expected from the traditional mass-loss rate and a model of the wind opacity at 15 Å, is  $\tau_* \approx 8$ . The best-fit model with  $\tau_* = 8$  is shown in Fig. 5. This model does not provide a good fit, having  $\Delta C = 108$ .

After fitting the non-porous, Owocki & Cohen (2001) line profile model, we next fit a given emission line with the model that includes porosity from spherical clumps (Owocki & Cohen 2006), as given by equation 9. This introduces an additional free parameter,  $h_\infty$ . We repeat the process described above, finding the best-fit model by adjusting the free model parameters to minimize the fit statistic, assessing confidence limits on parameters individually, and then examining joining confidence limits on pairs of parameters.

For the Fe XVII line at 15.014 Å, we found that  $h_\infty = 0.0$  was the best-fit value of the terminal porosity length. This is equivalent to a model without porosity (which is why we do not show a figure for this fit). The 68 percent confidence limit on this value is  $h_\infty = 0.43 R_*$  and the 90 percent confidence limit is  $h_\infty = 1.07 R_*$ . We can examine how this parameter interacts with the optical depth parameter,  $\tau_*$ . In Fig. 6 we show the confidence contours in two-dimensional  $h_\infty$ ,  $\tau_*$  parameter space. The correlation seen here between  $h_\infty$  and  $\tau_*$  arises from the ability of porosity to reduce the effective opacity of the wind, by hiding atomic opacity in optically thick clumps. And just as is expected theoretically (Owocki & Cohen 2006), the effect only becomes significant once the porosity length is equivalent to the local radius (here, roughly 1.5  $R_*$  and above, based on the fitted value of  $R_o$ ). The confidence contours, enclosing parameter values

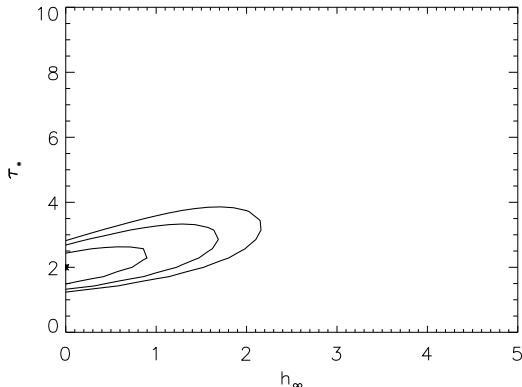


**Figure 5.** The Fe XVII line at 15.014 Å in the MEG (top) and HEG (bottom), with the best-fit non-porous model having  $\tau_* = 8$  superimposed. This model fit is statistically unacceptable.

that provide acceptable fits, show increasing correlation as  $h_\infty$  increases, but the effect of porosity on  $\tau_*$  does not become significant until  $h_\infty$  is above 1  $R_*$ . The strong effect of large  $h_\infty$  values and this parameter's strong correlation with  $\tau_*$  can be better seen in fits to noisier lines, with weaker constraints. The contours extend to higher  $h_\infty$  and  $\tau_*$  values, and start to curve upward for  $h_\infty > 1 R_*$ , as can be seen in §4.

We have already shown that models with  $\tau_* = 8$ , the value implied by the traditional mass-loss rate, provide poor fits to this line. And even the 95 percent confidence region in the porous model fitting does not enclose any models with  $\tau_* = 8$ . However, we can still ask how large a value of  $h_\infty$  is required to accommodate this high value of  $\tau_*$  expected from the traditional mass-loss rate. When we fit a model with  $\tau_* = 8$  fixed and porosity included to reduce the effective optical depth of the wind, we find a best-fit value for the terminal porosity length of  $h_\infty = 3.64 R_*$ . We show this high  $\tau_*$ , high  $h_\infty$  model in Fig. 7. Although this model cannot be rejected outright, it provides a worse fit to the data than does the non-porous model. The  $\Delta C$  between these two models is  $\Delta C \approx 15$ , indicating that the non-porous model is preferred at the 99.9 percent confidence level. In other words, if the best-fit non-porous model is the correct model that completely describes the data, then there is only a 0.1 percent chance of obtaining a fit as poor as (e.g. with the same C statistic as) the one provided by the best-fit porous

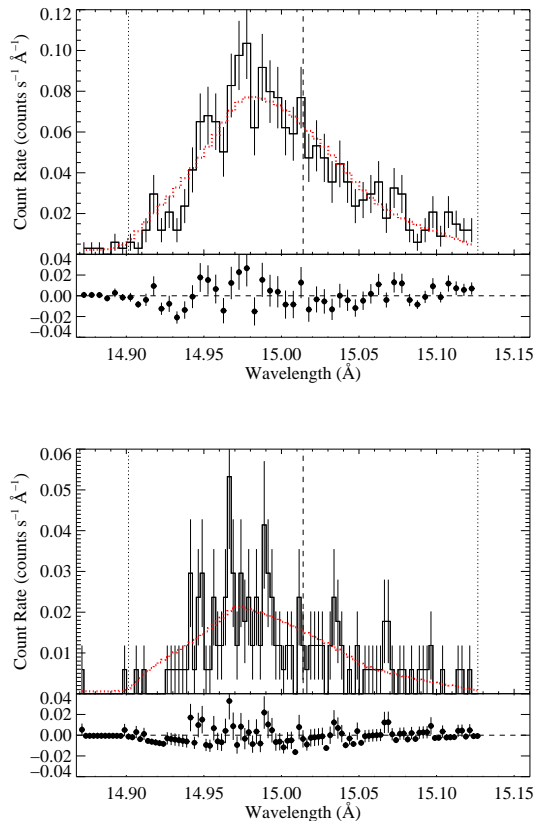




**Figure 6.** Confidence contours (68, 90, and 95 percent) for the porous model fitting of the the Fe xvii line at 15.014 Å. This porous model assumes spherical clumps, and the confidence contours show the joint distribution of  $h_\infty$  and  $\tau_*$ . Note that we use the same axes ranges for all subsequent  $h_\infty, \tau_*$  confidence plots in this paper in order to facilitate easy comparisons.

model due to random error. This model has a noticeable bulge on the extreme blue wing as well as one near line center, which is where the agreement is the worst; in both places, there are many consecutive bins for which the model lies above the data. These two morphological features are characteristic of porous models and ultimately prove to be the cause of the porous models’ inability to provide better fits to the data than the non-porous models.

After fitting the porous model with spherical clumps, determining the best-fit model parameters, assessing the parameter confidence limits, and examining a model for which  $\tau_*$  is fixed at the value implied by the traditional, literature mass-loss rate, we repeat the entire process for the porous model that assumes flattened clumps, as given by equation ???. For the representative Fe xvii line at 15.014 Å, we find a best-fit  $h_\infty = 0.0$ , just as we did for the porous model with spherical clumps. In other words, the non-porous model is preferred over the porous model with flattened clumps (and again, we do not bother to show this model, because it is functionally identical to the one shown in Fig. 3). When we test the confidence limits on the model parameters, we find the same limits we found for the non-porous model fits, at 68 percent confidence:  $1.63 < \tau_* < 2.35$ ,  $1.38 < R_o/R_* < 1.65$ , and  $5.04 \times 10^{-4} < \text{norm} < 5.51 \times 10^{-4}$ . This means that allowing for porosity from flattened clumps does not improve the fit quality for any combination of other parameters. When we evaluate the confidence limits on  $h_\infty$  itself, we find a 68 percent upper limit of only  $h_\infty = 0.04 R_*$ , and a 90 percent upper limit of  $0.10 R_*$ . So, even modest values of the terminal porosity length in the context of porosity from flattened clumps make the fits significantly worse. In Fig. 8 we show the joint  $h_\infty, \tau_*$  confidence limits. And in Fig. 9 we show the best-fit porous model assuming flattened clumps but with the optical depth parameter,  $\tau_*$ , fixed at the value implied by the literature mass-loss rate,  $\tau_* = 8$ . Even this best-fit model provides a poor fit to the data – much worse than that provided by the porous model with spherical clumps.

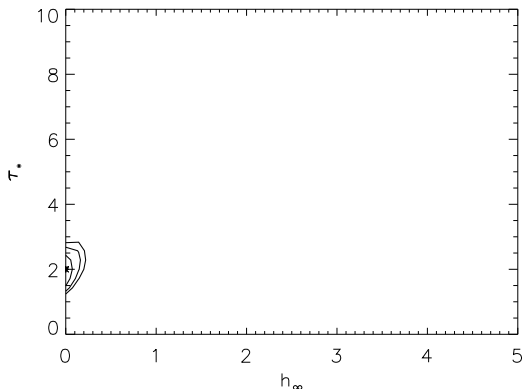


**Figure 7.** The Fe xvii line at 15.014 Å in the MEG (top) and HEG (bottom), with the best-fit porous model (with spherical clumps) having  $\tau_* = 8$  superimposed. Compare to Fig. 3.

## 4.2 Sensitivity of fitting results to modeling assumptions

We have made various assumptions and choices in carrying out the line-profile modeling, as described in the previous subsection. And we therefore have investigated many of these, again using the Fe xvii line at 15.014 Å as a test case. In this subsection, we report on the sensitivity of our results to the following assumptions and choices: background subtraction; determination of the continuum level; exclusion of portions of the line due to possible blending; inclusion of the weak HEG data; the adopted values of  $\beta$  and  $v_\infty$  for the wind; whether to allow the X-ray volume filling factor to vary with radius (as parameterized by  $q$  in  $f \propto r^{-q}$ ); and for porous models, the form of the bridging law that described how the effective opacity behaves as optically thin clumps make the transition to optical thickness. We will very briefly describe those factors that we found to be unimportant, and discuss in a little more detail those that did make a difference. The base-line model fitting we describe is the non-porous modeling described in the previous subsection, except that we fit only the MEG data (and thus are able to test the effects of including the HEG data).

We examined the default background spectra, which were very low, and also experimented with fitting the 15014 Å line with and without the background subtracted and found almost no difference. We therefore opt to neglect the

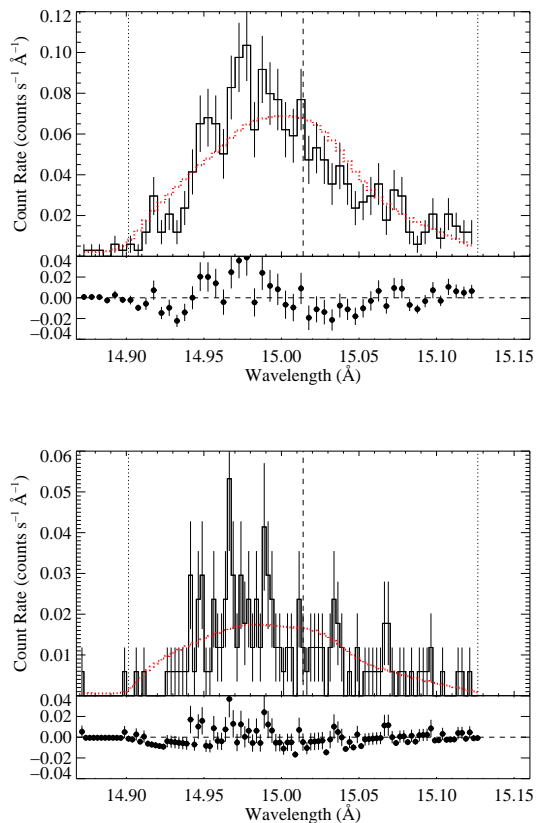


**Figure 8.** Confidence contours (68, 90, and 95 percent) for the porous model fitting of the the Fe xvii line at 15.014 Å. This porous model assumes oblate, or flattened, clumps, and the confidence contours show the joint distribution of  $h_\infty$  and  $\tau_*$ .

background when fitting each of the lines in the spectrum. The sensitivity to the continuum fit is a little greater, but still nearly negligible. When we changed the continuum level by a factor of two, none of the parameter values changed by more than ten percent. Some lines in the spectrum are blended with weaker lines. The cleanest way to handle this situation is to exclude the contaminated bins from the modeling. To test the effects of this, we eliminated 0.03 Å from the red wing of the 15.014 Å line and refit the data. We then repeated this experiment eliminating 0.07 Å - leaving only about two-thirds of the data. Even in this second, extreme case, the fit parameters varied by less than ten percent and the confidence regions only expanded slightly.

For most lines, the HEG data is significantly weaker than the MEG data. We find for the 15.014 Å line that including the HEG data changes the best-fit model parameters by, at most, a few percent, but it does tighten the confidence limits somewhat. The effect of including the HEG data is somewhat more significant for the shorter wavelength lines, where the effective area of the HEG is larger. There is very little penalty for including the HEG data, so we do for all lines shortward of 16 Å. We also fit the MEG and HEG data separately for the 15.014 line to verify that there are not systematic effects between these two spectra; and there are not. The separate fits give results that are very similar to each other, with significantly overlapping 68 percent confidence limits for all parameters.

The original Owocki & Cohen (2001) line profile model allows for a radially varying filling factor of X-ray emitting plasma, parameterized as a power law function of radius. Values of the power-law index,  $q$ , that differ significantly from zero (no radial variation) can cause changes in the line profiles that are not insignificant, effectively weighting the emission from parts of the wind according to their velocity (via the beta-velocity law relationship between velocity and radius). However, we find that when we allow  $q$  to be a free parameter the best-fit value is generally very close to zero. For the representative 15.014 line, it is  $q = -0.09$ , and  $q = 0$  is included in the 68 percent confidence region. The general result is consistent with that found by other investigators



**Figure 9.** The Fe xvii line at 15.014 Å in the MEG (top) and HEG (bottom), with the best-fit porous model (with flattened clumps) having  $\tau_* = 8$  superimposed. Compare to Fig. 3 and Fig. 7.

for this and other stars. Thus, to keep the number of free parameters manageable, we fix  $q = 0$ .

The factors discussed above have a very minor influence on the results of the line fitting. However, the remaining factors can have a significant effect.

The velocity-law exponent,  $\beta$ , affects line profiles for two reasons: (1) the velocity law affects the mapping between radius and Doppler shifted wavelength, and so affects the emission profile; and (2) via the continuity equation, it affects the density and so affects both the emission and the absorption. Indeed, for our representative emission line, when we change the value of  $\beta$  from 1 to 0.8, both  $\tau_*$  and  $R_0$  change by 10 to 20 percent. The determinations of  $\beta$  for  $\zeta$  Pup vary from at least 0.9 to 1.15, and so using a value of  $\beta = 1$  seems reasonable, especially as it speeds the calculation of the line profile model by allowing the optical depth integral to be done analytically, so we will use that value. If, in the future, a new and more accurate determination of  $\beta$  is made, and it differs significantly from  $\beta = 1$ , then the results reported in this paper can be scaled accordingly. We also note that the X-ray emitting plasma and the bulk wind that attenuates the X-rays may not necessarily be described by the same beta velocity law. However, there is no independent evidence for this, and with the short post-shock cooling lengths expected in the relatively dense wind of  $\zeta$  Pup the X-ray emitting plasma in the wind is more likely to have a

velocity close to the ambient wind velocity<sup>4</sup>. And furthermore, the observed X-ray emission line widths in  $\zeta$  Pup and other early O supergiants is completely consistent with the  $\beta$  and  $v_\infty$  values inferred from UV and optical spectroscopy of these stars.

The terminal velocity of  $\zeta$  Pup is relatively well established, with reasonable estimates from several different groups that vary by about  $\pm 10$  percent about our adopted value of  $2250 \text{ km s}^{-1}$ . However, when we explored the effect of varying the terminal velocity in our fitting of non-porous wind profile models to the  $15.014 \text{ \AA}$  line, we found that the value of  $\tau_*$  was quite sensitive to the assumed wind terminal velocity. This can be understood because the blue shift of the line centroid in the dimensionless, scaled wavelength parameter,  $x \equiv (\lambda/\lambda_o - 1)c/v_\infty$ , depends directly on the degree of wind absorption. The same observed profile appears more blue shifted in scaled wavelength units if the terminal velocity is (assumed to be) smaller. Our tests with the  $15.014 \text{ \AA}$  line show that the best-fit value for  $\tau_*$  ranges from 2.16 to 1.35 when we use terminal velocities between 2200 and  $2485 \text{ km s}^{-1}$ . This variation is larger than that caused by every other parameter uncertainty and assumption we have explored.

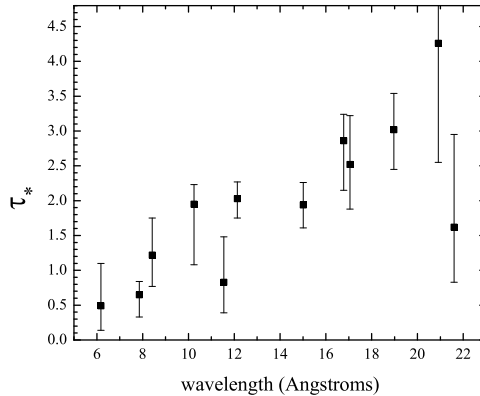
For porous models, there are at least two opacity “bridging laws” that span the limit between optically thin and thick clumps ( $\kappa_{\text{eff}} = \kappa$  to  $\kappa_{\text{eff}} = \ell^2/m_{\text{clump}}$ ), given by equations (4) and (6) in Owocki & Cohen (2006). We call these the “exponential” ( $\kappa_{\text{eff}}/\kappa = (1 - \exp(-\tau_c))/\tau_c$  and  $\kappa_{\text{eff}}/\kappa = (1/(1 + \tau_c))$ ) “Rosseland” bridging laws. We fit the  $15.014 \text{ \AA}$  line (both MEG and HEG) with (spherical clump) porosity models, using both bridging laws. We found a  $\approx 30$  percent difference in the terminal porosity length,  $h_\infty$ , required to fit the data when  $\tau_*$  was fixed at a high value, in the sense that the exponential form of the effective opacity requires larger porosity lengths and the Rosseland form requires smaller ones. The Rosseland model provides a modestly better fit than the exponential one. We use the Rosseland form throughout this paper for both types of porous model fits (spherical clumps and oblate clumps).

## 5 RESULTS OF THE LINE PROFILE MODEL FITTING

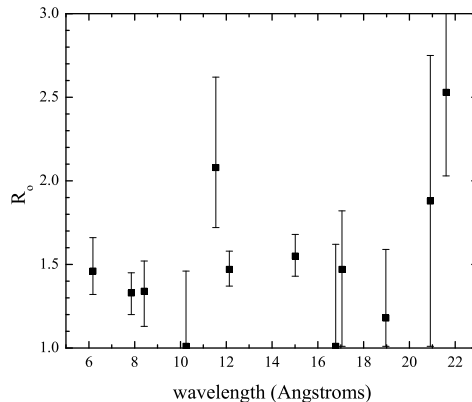
We repeated exactly the process described in the previous section for fitting non-porous and then two types of porous models to individual emission lines on every emission line in the spectrum of  $\zeta$  Pup, as indicated in Fig. 1. In cases where lines of relatively comparable strength are blended, we generally fit the multiple lines simultaneously. In several cases, including Ne x Ly $\alpha$ , we excluded a portion of the line because of possible blending with an iron line. For shorter wavelength helium-like line complexes, we fit the components simultaneously, with model parameters tied together (and the alteration of the profiles of the forbidden and intercombination lines by the UV photoexcitation of the excited hot ionself-consistently accounted for). For oxygen, though, where the spectral resolution is highest and the resonance scattering may affect the resonance line, we report fits for only the intercombination line. For several lines, including N VII Ly $\alpha$ , contamination, or blending by strong lines with uncertain relative intensities (N VI He $\beta$  in the case of N Ly $\alpha$ ) makes it impossible to reliably constrain the model parameters. Altogether we obtain reliable and interesting results from 14 emission lines or line complexes. Some of these do not provide meaningful constraints when we fit the non-porous models, and so we report on results for somewhat fewer lines in those cases.

### 5.1 Non-porous model fitting results

For the non-porous model fits, we list the best-fit parameters and confidence limits in Table 2 and graphically summarize the results for  $\tau_*$  and  $R_o$  in Figures 10 and 11. There appears to be a trend in the optical depth values, which will discuss in the following section. Here we show fitting results from three more important emission lines, which clearly demonstrate the correlation between  $\tau_*$  and wavelength. The short wavelength line with the tightest constraints in the Ly $\alpha$  line of Mg XII at 8.421 Å, which we show – both the MEG and HEG data along with the simultaneous best-fitting model – in Fig. 12. That figure also shows the jointly constrained  $\tau_*$  and  $R_o$  confidence limits. We show a similar figure for the longer wavelength O VIII Ly $\alpha$  line in Fig. 14.



**Figure 10.** Values of  $\tau_*$  derived from the non-porous model fits, shown with their 68 percent confidence limits. Line complexes and blends that were fit with multiple model components are represented by only one point.



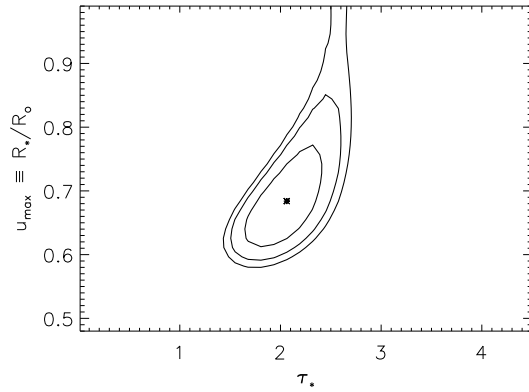
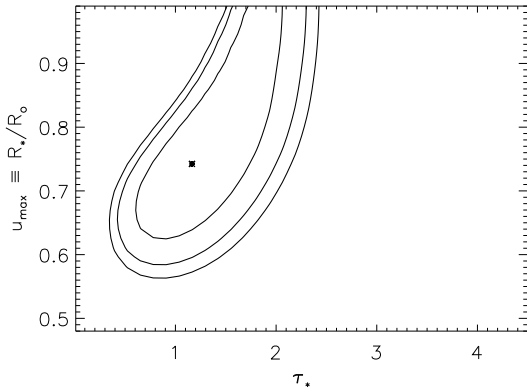
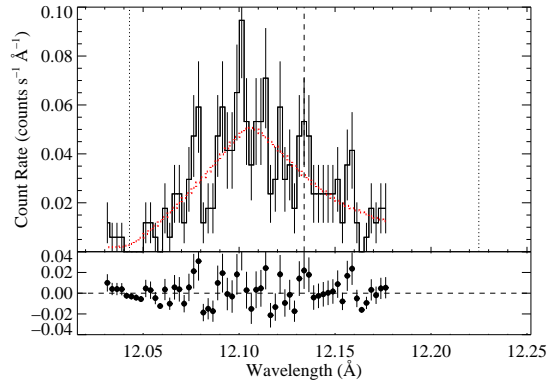
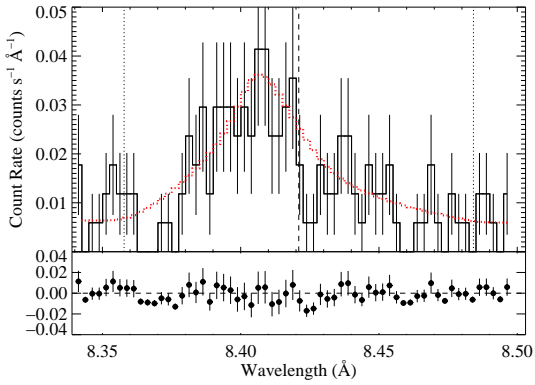
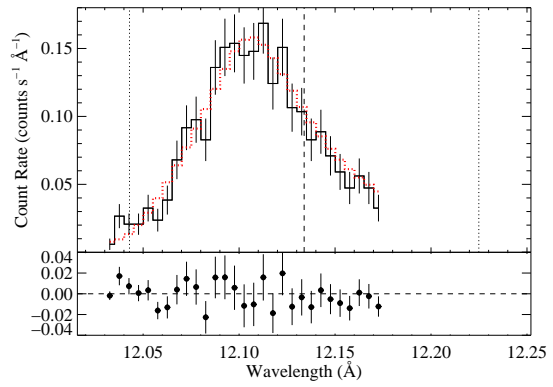
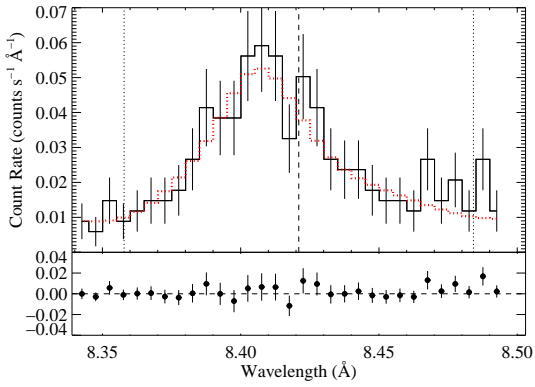
**Figure 11.** Values of  $R_o$  derived from the non-porous model fits, shown with their 68 percent confidence limits. Line complexes and blends that were fit with multiple model components are represented by only one point.

**Table 2.** Non-porous model fits

ion	wavelength (Å)	$\tau_*$	$R_o$ ( $R_*$ )	normalization ( $10^{-5}$ ph cm $^{-2}$ s $^{-1}$ )
Si XIV	6.1822	$0.49^{+.61}_{-.35}$	$1.46^{+.20}_{-.14}$	$0.77^{+.11}_{-.14}$
Mg XI	7.8503	$0.65^{+.19}_{-.32}$	$1.33^{+.12}_{-.13}$	$1.33^{+.17}_{-.13}$
Mg XII	8.4210	$1.22^{+.53}_{-.45}$	$1.34^{+.18}_{-.21}$	$2.95^{+.24}_{-.24}$
Ne X	10.2388	$1.95^{+.28}_{-.87}$	$1.01^{+.45}_{-.00}$	$2.99^{+.31}_{-.29}$
Ne IX	11.5440	$0.83^{+.65}_{-.44}$	$2.08^{+.54}_{-.36}$	$5.00^{+.40}_{-.50}$
Ne X	12.1339	$2.03^{+.24}_{-.28}$	$1.47^{+.11}_{-.10}$	$26.9^{+1.1}_{-.7}$
Fe XVII	15.014	$1.94^{+.32}_{-.33}$	$1.55^{+.13}_{-.12}$	$52.4^{+2.5}_{-1.6}$
Fe XVII	16.780	$2.86^{+.38}_{-.71}$	$1.01^{+.61}_{-.00}$	$23.1^{+1.9}_{-1.2}$
Fe XVII <sup>a</sup>	17.051, 17.096	$2.52^{+.70}_{-.64}$	$1.47^{+.35}_{-.46}$	$17.2^{+0.9}_{-1.1}, 15.5$
O VIII	18.969	$3.02^{+.52}_{-.57}$	$1.18^{+.41}_{-.17}$	$37.0^{+2.8}_{-2.6}$
N VII	20.9099	$4.26^{+2.28}_{-1.71}$	$1.88^{+.87}_{-.87}$	$14.8^{+2.3}_{-1.9}$
O VII <sup>b</sup>	21.602, 21.804	$1.62^{+1.33}_{-.79}$	$2.53^{+.85}_{-.50}$	$31.9^{+4.0}_{-3.9}, 28.0^{+2.9}_{-3.8}$

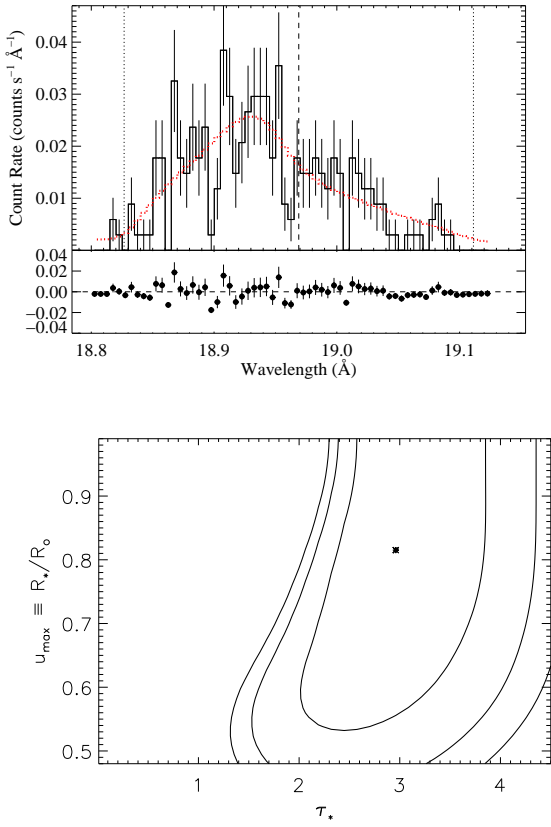
<sup>a</sup> We fit these two blended lines simultaneously, with a fixed normalization ratio of 0.9. Both line profile components were forced to have the same  $\tau_*$  and  $R_o$  values.

<sup>b</sup> In this case, while the two components'  $\tau_*$  and  $R_o$  parameter values were tied together, the two normalizations were free to vary independently.

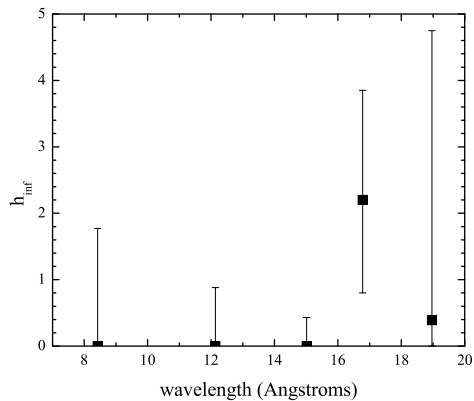


**Figure 12.** The MEG (top) and HEG (middle) measurements of the Mg XII Ly $\alpha$  line at 8.421 Å. This line shows a relatively small degree of blue shift and asymmetry, indicative of a low  $\tau_*$  value, as is expected at short wavelengths, where the wind opacity is smaller. We also show the 68, 90, and 95 percent confidence limits in  $\tau_*$ ,  $u_o$  parameter space (bottom).

**Figure 13.** The MEG (top) and HEG (middle) measurements of the Ne x Ly $\alpha$  line at 12.134 Å. This line shows an intermediate degree of blue shift and asymmetry, indicative of an intermediate  $\tau_*$  value, as is expected at its wavelength, where the wind opacity is larger than at the wavelength of the Mg XII Ly $\alpha$  line, but not as large as at longer wavelengths. We also show the 68, 90, and 95 percent confidence limits in  $\tau_*$ ,  $u_o$  parameter space (bottom).



**Figure 14.** The MEG (top) measurements of the O VIII Ly $\alpha$  line at 18.969 Å. This line shows a relatively large degree of blue shift and asymmetry, indicative of a higher  $\tau_*$  value, as is expected at longer wavelengths, where the wind opacity is larger. We did not include the very weak HEG data in the analysis of this line. We also show the 68, 90, and 95 percent confidence limits in  $\tau_*$ ,  $u_o$  parameter space (bottom).



**Figure 15.** For the emission lines where a meaningful constraint can be put on the porosity length from the model fitting (that assumes spherical clumps), we show the best-fit and 68 percent confidence limits on the terminal porosity length,  $h_\infty$ , in units of  $R_*$ .

## 5.2 Spherical clump porous model fitting results

For the fits of porous models that assume spherical clumps, we generally find – as we did for the demonstration line at 15.014 Å – that porosity lengths of zero ( $h_\infty = 0$ ) are preferred. The only exceptions are for a small number of low signal-to-noise lines, and in every one of these cases, porosity lengths of zero are allowed at the 68 percent confidence limit. However, of course some degree of porosity generally is also consistent with each of these lines. So, we can still find out some information from these fits. In this subsection, we show the 68 percent upper limits to the terminal porosity length parameter,  $h_\infty$ , for each line that is strong enough to warrant fitting with these models. We also examine the trade-off between  $h_\infty$  in  $\tau_*$ , both by looking at the joint probability distributions of these two key parameters, and by fitting models to several of the stronger lines for which we fix the value of  $\tau_*$  at that implied by the traditional, unclumped mass-loss rate of  $8.3 \times 10^{-6} M_\odot \text{ yr}^{-1}$ .

In Fig. 15 we show the best-fit values of  $h_\infty$  and their individual 68 percent confidence limit error bars. Note that for the line at 16.78 Å, the value of  $h_\infty = 0$  is easily enclosed by the 90 percent confidence limits (and even by the 68 percent confidence limits, when they are jointly computed between  $h_\infty$  and  $\tau_*$  as shown in the third panel on the left in Fig. 16.

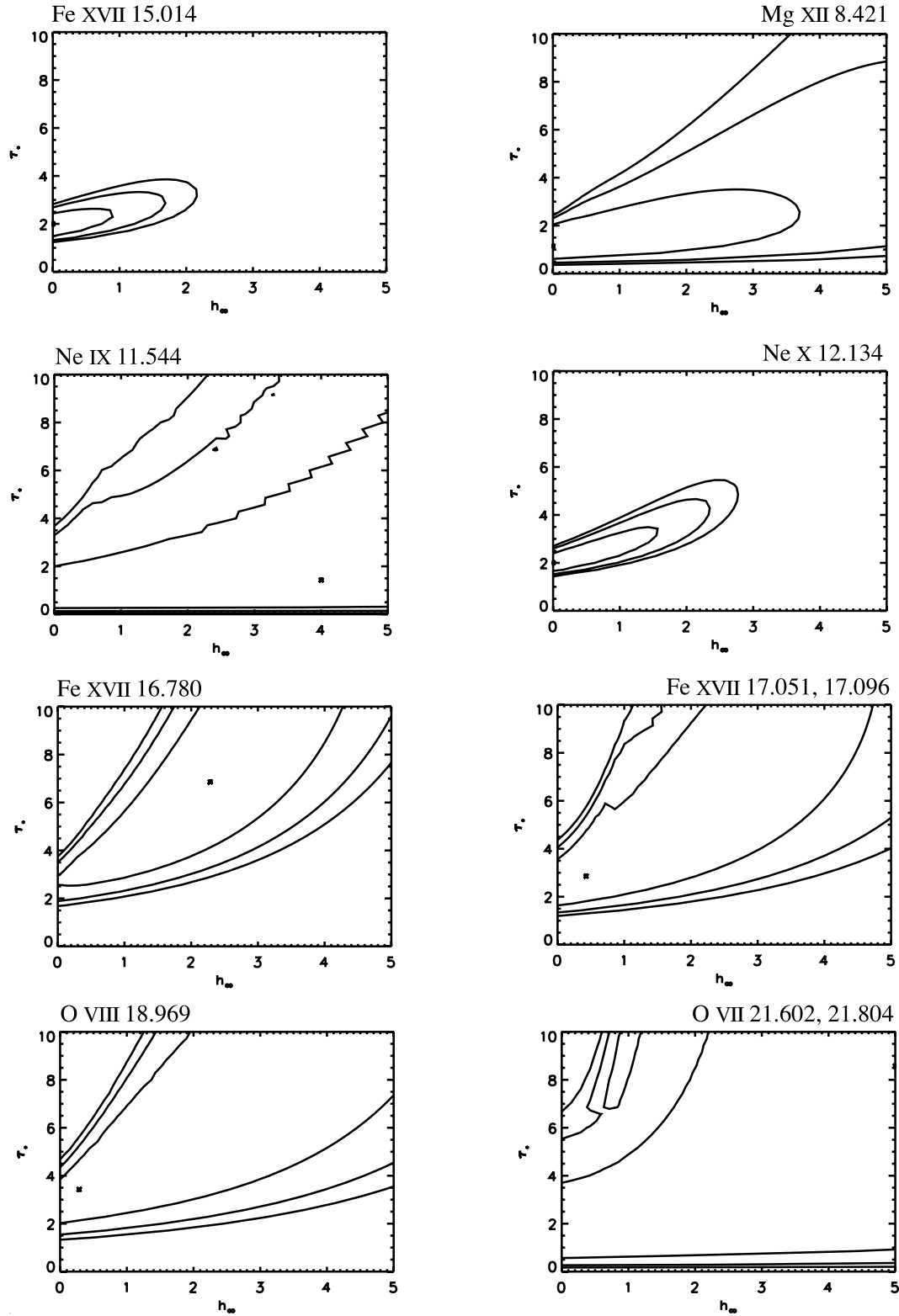
In Fig. 16 we show the joint confidence limits on  $h_\infty$ ,  $\tau_*$  for eight lines. Only the Fe XVII line at 15.014 Å, which we have shown already in the previous section, the Ne X Ly $\alpha$  line at 12.134 Å, and the Mg XII Ly $\alpha$  line at 8.421 Å provide relatively tight constraints on  $h_\infty$  and  $\tau_*$ , ruling out values of  $\tau_*$  that are high enough to be consistent with the unclumped literature mass-loss rate. The other, weaker or blended lines provide more modest constraints, and generally allow for any value of  $h_\infty$ , provided  $\tau_*$  is high enough. In each case, the non-porous model ( $h_\infty = 0$ ) cannot be ruled out at even the 68 percent confidence limit.

These confidence regions in the  $h_\infty$ ,  $\tau_*$  parameter space enable us to quantitatively examine the trade-offs between

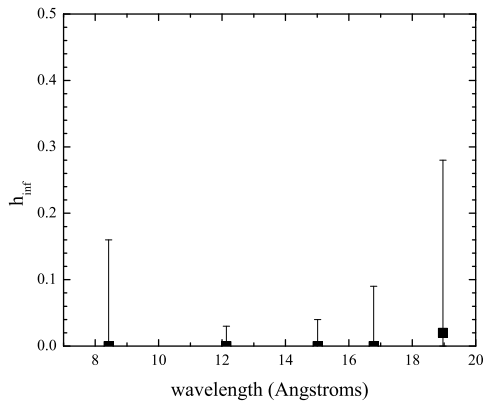
the two parameters and, thus, between lower mass-loss rates and significant wind porosity. We have already shown in the previous section that for the Fe XVII line, the value of the fiducial optical depth implied by the traditional mass-loss rate,  $\tau_* = 8$  is not only disfavored compared to models with little or no porosity, (i.e. it lies outside the 95 percent confidence region) but that such high optical depth models require very large porosity lengths ( $h_\infty = 3.3 R_*$ , for this line). The same is true of the other higher signal-to-noise, unblended lines. We can assess this quantitatively by noting the minimum value of  $h_\infty$  for which a given confidence level intersects the  $\tau_*$  value implied by the assumed mass-loss rate. Given the wind opacity model we discuss in the next section, the traditional mass-loss rate of  $\zeta$  Pup implies a value of  $\tau_* = 4$  for the Ne X line at 12.134 Å, and a value of  $\tau_* = 10$  for the O VIII line at 18.969 Å, for example. The 90 percent confidence limit contour intersects these values of  $\tau_*$  at  $h_\infty \approx 1.3 R_*$  for both of these lines. Even lines that are quite poorly constrained overall, such as the Fe XVII lines at 16.780 Å and the blend near 17 Å, require terminal porosity lengths of  $h_\infty > 1 R_*$  for model fits that are adequate.

We also fit high  $\tau_*$  models, with that parameter fixed at the value implied by the traditional mass-loss rate. For Mg XII at 8.421 Å, we find that  $h_\infty = 3.14$  for  $\tau_* = 4$ , with 68 percent confidence limits of 1.67 and 5.58. This result is very similar to that which we obtained for the Fe XVII line at 15.014 Å. The upper end of the  $h_\infty$  distribution is not very meaningfully constrained. A porosity length of 5 implies that there are just a few, very small and dense clumps in the entire wind! *Note to coauthors: We still have to calculate a few more of these.*





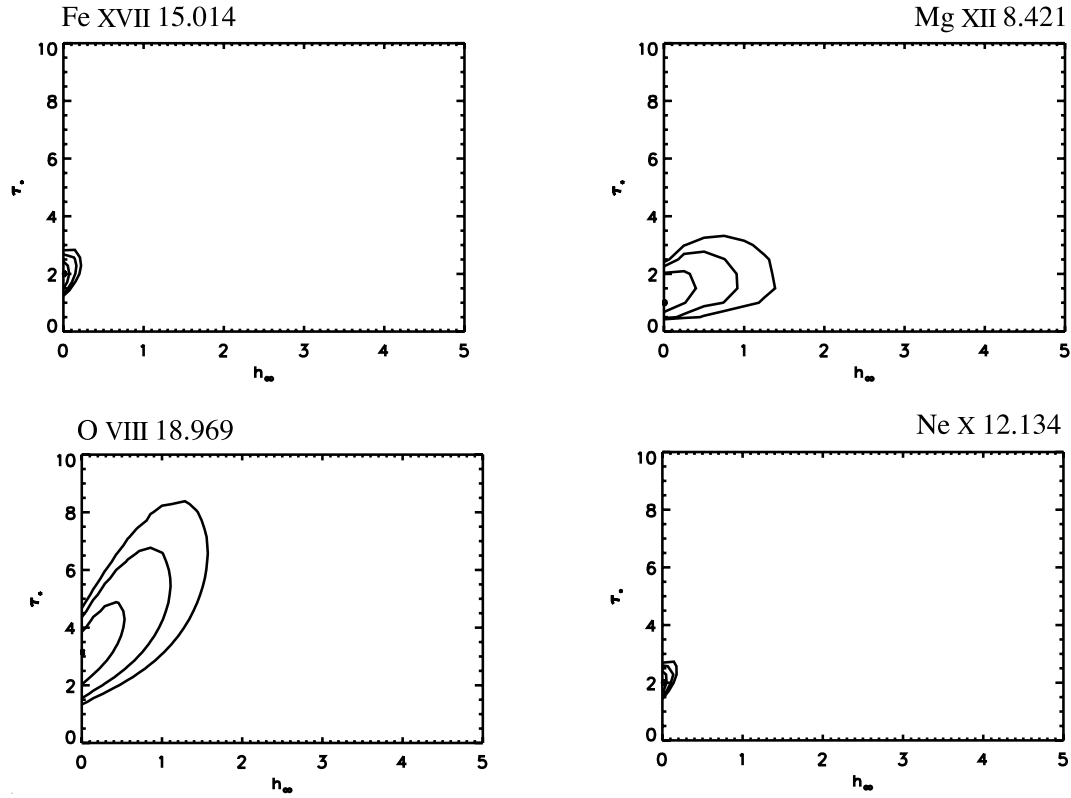
**Figure 16.** The joint  $h_\infty, \tau_*$  68, 90, and 95 percent confidence regions for fits of porous models assuming spherical clumps to the strongest lines in the *Chandra* spectrum of  $\zeta$  Pup. The best-fit models are indicated by the asterisks. Note that for the higher signal-to-noise lines, a value of  $h_\infty = 0$  tends to be preferred, although significantly larger values are allowed. And for the lower signal-to-noise lines, even if higher values are marginally preferred, a value of  $h_\infty = 0.00$  is allowed at the 68 percent confidence limit in every case.



**Figure 17.** For the emission lines where a meaningful constraint can be put on the porosity length from the model fitting (that assumes oblate clumps), we show the best-fit and 68 percent confidence limits on the terminal porosity length,  $h_{\infty}$ , in units of  $R_*$ .

### 5.3 Oblate clump porous model fitting results

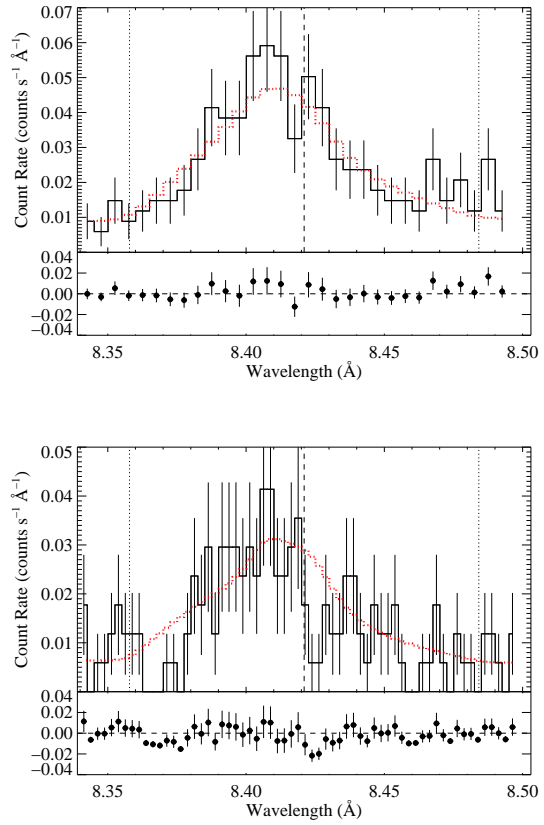
We perform similar tests on the data using the porous model that assumed oblate clumps. In general, we find even worse fits than for the spherical clump porosity modeling. In every case but one, porosity lengths of zero are preferred, and in the other case, a value of  $h_{\infty} = 0.02 R_*$  gives the best fit, but any value on  $0.0 < h_{\infty} < 0.28 R_*$  is allowed at the 68 percent confidence level. Even modest values of the porosity length are rejected in the case of oblate clumps, as is shown in Figs. 17 and 18. The lines we do not show in these plots did not provide any meaningful constraints on the porosity length.



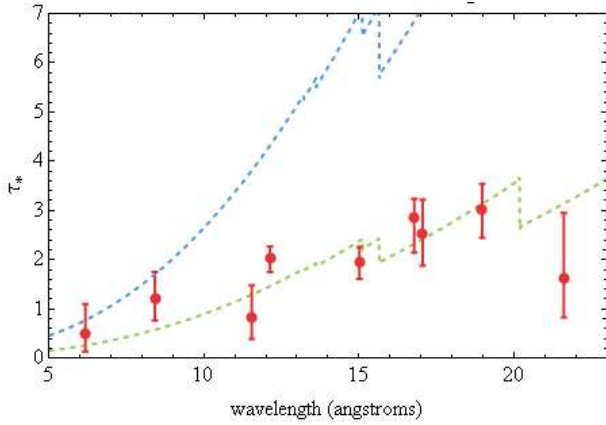
**Figure 18.** The joint  $h_{\infty}, \tau_*$  68, 90, and 95 percent confidence regions for fits of porous models assuming oblate clumps to the strongest lines in the *Chandra* spectrum of  $\zeta$  Pup. The best-fit models are indicated by the asterisks. Note that the constraints are much tighter than for the porous models that assume spherical clumps. In every case,  $h_{\infty} = 0$  is preferred.

We also show in Fig. 19 a fit to the Mg XII Ly $\alpha$  line of a model that includes porosity from oblate clumps and has the optical depth parameter fixed at  $\tau_* = 4$ , the value implied by the radiational mass-loss rate. This fit has a C statistic value that is higher than that given by the global best fit model (with a low  $\tau_*$  and  $h_\infty = 0$ ) by  $\Delta C \approx 8$ , for a significance of 99.7 percent.

In summary, the porous models do not provide significantly better fits to any of the data than do the non-porous models. And for the highest quality data, the porous models provide worse fits. This is especially true for the porous models based on oblate clumps. When porosity is allowed, the values of the porosity length that are required to provide adequate fits to the data are always in excess of  $h_\infty = 1 R_*$ .



**Figure 19.** Best-fitting porous model (assuming oblate clumps) with a high value of the wind optical depth ( $\tau_* = 4$ , as implied by the traditional mass-loss rate).



**Figure 20.** Values of  $\tau_*$  derived from the non-porous model fits, shown as points with error bars. The value of  $\tau_*$  expected from the literature mass-loss rate of  $8.3 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$  is shown in blue. Treating the mass-loss rate as a free parameter, the best fit value of  $3.0 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$  is shown in green.

## 6 DISCUSSION AND IMPLICATIONS OF RESULTS

**7 CONCLUSIONS**

**ACKNOWLEDGMENTS**

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