Astronomy 16 – Modern Astrophysics

Fall 2014
Homework 5
due: Tuesday, October 28, in class

It is very important to present your solutions neatly and clearly. Use units when appropriate, state where your numbers come from, explain what you're doing. Use sketches when you think they'd be useful.

1. **Convection**: Let's derive the criterion that determines whether convection transports energy at a given location in the star. Consider two cylindrical parcels of material, each of the same height *dr* and surface area *dA* and at the same radial location within the star. The unprimed parcel (with properties T, ρ, etc.) is in hydrostatic equilibrium, and can be considered the "average" stellar material. The primed parcel (with properties T', ρ', etc.) may not be in equilibrium and is like a bubble. Both are at the same location in the star, so they are surrounded by the same material.

   (a) Use Newton's second law to derive an equation for the forces acting on the unprimed parcel, which is at rest. Your expression should be in terms of P_{top} and P_{bottom}, the pressures on the top and bottom, respectively; the density ρ, the surface area on top or bottom dA; $M_r(r)$, the mass interior to radius r; the gravitational constant G; and radius r. Carefully label forces on the parcels in a diagram and make sure all the negative signs are in the right places in your equation.

(b) Use Newton's second law to derive an equation for the forces acting on the primed parcel (the "bubble"), which may have a non-zero acceleration.

(c) Combine (a) and (b) to find an expression for acceleration in terms of G, $M_r(r)$, ρ, ρ', and r; simplify your expression as much as possible. Under what conditions will the primed parcel rise, sink, or remain in place?

(d) Convert your condition in part (c) to a condition on the temperature in the unprimed and primed parcels (assume the perfect gas law holds). What does this condition imply about the direction of energy transfer when convection occurs?

(e) So far, you have only determined the condition under which the primed parcel will *instantaneously* accelerate up or down or remain fixed. But the more important question is the following: Presume that the primed parcel is indeed perturbed so as to instantaneously accelerate upwards, in accord with part (c). Once above its original location, will the primed parcel continue to accelerate upwards or will it stop and sink back toward equilibrium? Clearly, as the bubble rises, the density and pressure surrounding it will decrease. The density of the bubble will also change as it rises. However, they need not vary at the same rate. Show that if the radial density gradient of the bubble is steeper than the radial density gradient of its surroundings, then once the bubble *begins* to rise in accord with the condition derived in part (c), it will continuously accelerate upward. Convection is an efficient energy transfer mechanism in this case. Conversely, if the radial density gradient of the surroundings is steeper than the density gradient of the bubble, show that the bubble will *begin* an upward journey if condition (c) is satisfied, but it will soon stop. In this case, the energy cannot be transported by convection. Hint: a graph of density vs. radius for primed and unprimed parcels will be extremely helpful.

These next two problems are about the *magnitude* system. Note that the first of these has two parts.

2. Ryden & Peterson, problem 14.1 (p. 349). There's a table in the appendix that might be helpful. Also answer this question: If the star's distance from us were to double, what would its apparent V magnitude be?

3. Ryden & Peterson, problem 14.7 (p. 349).