

Astronomy 16 Modern Astrophysics

Fall 2014

Homework 7

due: Wednesday, December 3, by 5 PM

It is very important to present your solutions neatly and clearly. Use units when appropriate, state where your numbers come from, explain what you're doing. Use sketches when you think they'd be useful and when a sketch is asked for, make sure you provide the level of detail that's requested. Finally, make sure you answer all the parts of each problem, including my supplemental questions for some of the book problems.

1. Ryden & Peterson problem 18.5 (p. 431). Note that 10 km/s is the typical interstellar medium sound speed, so when the supernova blast wave slows to this speed, it no longer is a shock wave and thus no longer perturbs and heats the interstellar gas. After answering both parts of problem 18.5, please also answer the following: (i) what is the radius of the supernova "bubble" by the time the blast wave has slowed to the sound speed? (ii) Estimate the number of stars that are contained within this supernova "bubble" based on the typical inter-star distance in the Milky Way, and (iii) what would the distance have to be to the supernova if the bubble/remnant at the time it has become sub-sonic if it subtends an angle on the sky of half a degree (equivalent to the size of the full moon)? Finally, (iv) for part (b) you assumed that energy is conserved. But there are some other forms of energy, in addition to kinetic energy, that the energy could take on during the expansion phase described in part (b). What are some of them? And if there are energy losses, will the bubble be bigger or smaller than your calculation?

2. Ryden & Peterson problem 18.7 (p. 431). You should make your plot by hand, but make it neat and quantitatively accurate.
3. Ryden & Peterson, problem 16.9 (p. 392).
4. For the cloud in problem 3, calculate how small it would have to get (its diameter) if it started to collapse under its own gravity before it became optically thick to light. Assume that the atoms or molecules in the cloud have an average cross section of 10^{-28} m^2 per particle at the wavelength of that light. You can assume you're looking right through the middle of the cloud and that the density is uniform within the cloud.
5. Consider an interstellar cloud that's about to start collapsing to form a star or stars. It is spherical, has a radius of 0.1 pc, and has a mass of 1 solar mass. It is also rotating, which you can measure via the Doppler shift. Its rotation is more or less like a rigid body, with a constant angular velocity of $\omega = 3.2 \times 10^{-14}$ radians/second.
 - a. If angular momentum is conserved as this cloud collapses to form a one solar mass, one solar radius, sun-like star, what with the rotational velocity (in km/s) be at the equator of the resulting star? Note that the orbital angular momentum of a point mass undergoing uniform circular motion is just mvr , mass times velocity times radius. For a distribution of matter, like a sphere, the expression is modified by a constant factor, of order unity. For a sphere with uniform density the angular momentum is $\frac{2}{5} mvr$, where v is the velocity at the sphere's equator.
 - b. There is an upper limit to the rotational velocity of an object, as the centrifugal force will cause a rapidly rotating

object to break apart if its magnitude exceeds that of the force holding the object together. For a star or planet, that force is gravity. Given that centrifugal force (which is the “fictitious force” associated with centripetal acceleration) is given by $F_c = mv^2/r$, find the maximum equatorial rotational velocity of a sun-like star, in km/s. All the extra rotational velocity and associated angular momentum in your answer to (a), in excess of this amount you’ve found in (b), must be dissipated or stored somewhere, somehow.

- c. We know from the textbook, that the infalling material will form a disk around the star as a sort of reservoir for all that extra angular momentum. But the disk disappears after about ten million years and the star is born (so to speak). And the star can’t be spinning faster than the “break-up velocity” you computed in (b), so it still may have too much angular momentum. Maybe it can be stored in the orbit of a binary system. And the cloud can form two 0.5 solar mass stars rather than a single one solar mass star. Derive an expression for the angular momentum of the two half-solar-mass stars in orbit around their common center of mass. Your expression should be a function of the semi-major axis (note: the sum of the radii of each star’s orbit is the semi-major axis), and the star masses, but not the period or the velocity. You can assume a circular orbit. Note that Kepler’s third law must be obeyed.
- d. Evaluate your result from part (c) to determine what value of the semi-major axis would be required in order for the binary system’s orbital motion to contain even just a tenth of the original angular momentum of the cloud. Take the cloud’s angular momentum to be $2/5 mvr$.